Distributionally Robust Optimization

Singapore University of Technology and Design

Summer 2019 Graduate Course

Course Description

This is a graduate level special topics in optimization course which will focus on applications and methods to solve optimization problems under uncertainty - the main focus will be on distributionally robust optimization (DRO) where the decision-maker chooses an optimal decision accounting for the worst-case distribution that arises from a set of distributions. The goal is to get students introduced to the topic through the lens of two stage linear and discrete optimization problems. From a tools perspective, the students will pick up ideas in modeling, a better understanding of when the problems are easy or hard to solve and practical methods to solve such problems using techniques from linear optimization, conic optimization, integer optimization. The course is positioned as a research topics class. Students are expected to have taken a prior graduate level course on linear and discrete optimization and an advanced level undergraduate course in probability. Since this topic is currently an active area of research with many new directions emerging, the topics covered in the class will not be an exhaustive list on the subject, but rather biased by the instructor's interests. The hope is to get students excited on this topic for possibly developing new models, theory and applications in their own research areas, while highlighting the relevant literature to access.

Course Assessment

Homework - 50%, Project - 50%

Instructor

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Schedule

Venue: Think Thank 12
Dates: May 22 (Wed 2-5 pm), June 3 (Mon 10 am-1 pm), June 7 (Fri 2-5 pm), June 10 (Mon 10 am-1 pm), June 17 (Mon 10 am-1 pm), June 24 (Mon 10 am-1 pm), July 8 (Mon 10 am-1 pm), July 15 (Mon 10 am-1 pm), July 22 (Mon 10 am-1 pm), July 29 (Mon 10 am-1 pm), July 31 (Wed 2-5 pm), August 14 (Wed 2-5 pm)

Topics

(1) **Introduction to two stage stochastic linear and integer optimization** (class 1)

- Formulation and applications - newsvendor, portfolio optimization, facility location and transportation, PERT network, appointment scheduling
- Modeling issues
- Solution methods

References:

- *A. Shapiro, D. Dentcheva and A. Ruszczynski. Lectures on stochastic programming: Modeling and theory, 2nd edition, 2014, Chapter 1 and 2.*
- *G. B. Dantzig. Linear programming under uncertainty. Management Science, 1955.*
- *R J-B. Wets. Stochastic programs with fixed recourse: The equivalent deterministic program. SIAM Review 1983.*

(2) **Independent distributions** (class 2)

- Basics of computational complexity: P, NP, NP-hard, #P, #P-hard, reductions
- Hardness - Sums of random variables, network reliability, expected makespan in PERT network
- Easy instances - Series-parallel graphs with restricted support

References:

- *M. R. Garey and D. S. Johnson. Computers and intractability: A guide to the theory of NP-completeness, 1979.*
- *A. Schrijver. A course in combinatorial optimization, 2017, Chapter 6.*
- *J. S. Provan and M. O. Ball. The complexity of counting cuts and of computing the probability that a graph is connected. SIAM Journal of Computing, 1983.*
- *J. N. Hagstrom. Computational complexity of PERT problems. Networks, 1988.*
- *M. Dyer and L. Stougie. Computational complexity of stochastic programming problems. Mathematical Programming, Series A, 2006.*
- *R. H. Möhring. Scheduling under uncertainty: Bounding the makespan distribution. Computational Discrete Mathematics, 2001.*

(3) **Univariate marginals** (class 3 and 4)

- Ellipsoid method and equivalence of separation and optimization
- 0/1 optimization problems with univariate marginals: Primal and dual formulations
- Extensions to linear and integer optimization - extended formulations
- Exploiting symmetry in probabilistic combinatorial optimization - linear assignment

References:

- *M. Grötschel, L. Lovasz, A Schrijver. Geometric algorithms and combinatorial optimization, 1988.*
- *D. Bertsimas and J. N. Tsitsikilis. Introduction to linear optimization, 1997.*
- *M. M. Deza and M. Laurent. Geometry of cuts and metrics, 1997.*
- *I. Meilijson and A. Nadas. Convex majorization with an application to the length of critical paths. Journal of Applied Probability, 1979.*
- *W. K. K. Haneveld. Robustness against dependence in PERT: An application of duality and distributions with known marginals. Mathematical Programming Study, 1986.*
- *D. Bertsimas, K. Natarajan and C-P. Teo. Probabilistic combinatorial optimization: moments, semidefinite programming, and asymptotic bounds. SIAM Journal on Optimization, 2004.*
- *K. Natarajan, M. Song, C-P. Teo. Persistency model and its applications in choice modeling. Management Science, 2009.*
- *H-Y. Mak, Y. Rong and J. Zhang. Appointment scheduling with limited distributional information. Management Science, 2015.*
- *M. E. Dyer, A. M. Frieze, C. J. H. McDiarmid. On linear programs with random costs. Mathematical Programming, 1986.*

(4) **Comparing solutions: Independence and dependence** (class 5)

- Basics of submodular and supermodular set functions
- Price of correlation

References:

- *G. L. Nemhauser, L. A. Wolsey, M. L. Fisher. An analysis of approximations for maximizing submodular set functions-1. Mathematical Programming, 1978.*
- *L. Lovász. Submodular functions and convexity. Mathematical Programming: The state of art, 1983.*
- *G. Calinescu, C. Chekuri, M. Pál, and J. Vondrák. Maximizing a submodular set function subject to a matroid constraint. SIAM Journal on Computing, 2011.*
- *S. Agrawal, Y. Ding, A. Saberi and Y. Ye. Price of correlations in stochastic optimization. Operations Research, 2012.*

(5) **Multivariate marginals** (class 6)

- Nonoverlapping marginals
- Overlapping marginals - trees and chordal graphs

References:

- *C. Beeri, R. Fagin, D. Maier and M. Yannakakis. On the desirability of acyclic database schemes. Journal of the Association for Computing Machinery, 1983.*
- *M. J. Wainwright and M. I. Jordan. Graphical models, exponential families, and variational inference. Foundations and Trends in Machine Learning, 2008.*
- *L. Rüschendorf. Bounds for distributions with multivariate marginals. Stochastic Orders and Decision under Risk. IMS Lecture Notes - Monograph Series, 1991.*
- *X. V. Doan and K. Natarajan. On the complexity of non-overlapping multivariate marginal bounds for probabilistic combinatorial optimization. Operations Research, 2012.*
- *X. V. Doan, X. Li and K. Natarajan. Robustness to dependency in portfolio optimization using overlapping marginals. Operations Research, 2015.*

(6) **Moment problems and polynomial optimization** (class 8 and 9)

- Basics of semidefinite optimization
- Cone of nonnegative polynomials and sum of squares polynomials
- Moments cone
- Moment bounds with semidefinite optimization
- Application to steady-state performance analysis of queues

References:

- *M. Todd. Semidefinite optimization. Acta Numerica, 2001.*
- *M. Laurent. Sums of squares, moment matrices and optimization over polynomials. In Emerging Applications of Algebraic Geometry, Vol. 149 of IMA Volumes in Mathematics and its Applications, M. Putinar and S. Sullivant (eds.), 2009.*
- *A. W. Marshall and I. Olkin. Multivariate chebyshev inequalities. The Annals of Mathematical Statistics, 1960.*
- *J. B. Lasserre. Bounds on measures satisfying moment conditions. Annals of Applied Probability, 2002.*
- *P. A. Parrilo. Semidefinite programming relaxations for semialgebraic problems. Mathematical Programming Series B, 2003.*
- *D. Bertsimas and I. Popescu. Optimal inequalities in probability theory: A convex optimization approach. SIAM Journal on Optimization, 2005.*
- *J. B. Lasserre. A semidefinite programming approach to the generalized problem of moments. Mathematical Programming Series B, 2008.*
- *J. F. C. Kingman. Some inequalities for the queue GI/G/1, 1962.*
- *D. Bertsimas and K. Natarajan. A semidefinite optimization approach to the steady-state analysis of queueing systems. Queueing Systems, 2007.*

(10) **Moment based formulations** (class 10)

- Moment based formulations for distributionally robust optimization
- Basics of completely positive and copositive optimization
- Application to distributionally robust optimization

References:

- *H. Scarf, A min-max solution of an inventory problem. Studies in The Mathematical Theory of Inventory and Production. Stanford University Press, 1958.*
- *I. Popescu. Robust mean-covariance solutions for stochastic optimization. Operations Research, 2007.*
- *E. Delage and Y. Ye. Distributionally robust optimization under moment uncertainty with application to data-driven problems. Operations Research, 2010.*
- *D. Bertsimas, X. V. Doan, K. Natarajan, C-P. Teo. Models for minimax stochastic linear optimization problems with risk aversion. Mathematics of Operations Research, 2010.*
- *J. Goh and M. Sim. Distributionally robust optimization and its tractable approximation. Operations Research, 2010.*
- *S. Burer. On the copositive representation of binary and continuous nonconvex quadratic programs. Mathematical Programming, 2009.*
- *S. Burer. A gentle, geometric introduction to copositive optimization. Mathematical Programming, 2015.*
- *A. Berman and N. Shaked-Monderer. Completely positive matrices. World Scientific Publishing, 2003.*
- *K. Natarajan, C-P. Teo, Z. Zheng. Mixed 0-1 linear programs under objective uncertainty: A completely positive representation. Operations Research, 2011.*
- *W. Wiesemann, D. Kuhn and M. Sim. Distributionally robust convex optimization. Operations Research, 2014.*
- *Q. Kong, C-Y. Lee, C-P. Teo and Z. Zheng. Scheduling arrivals to a stochastic service delivery system using copositive cones. Operations Research, 2013.*

(11) **Robust linear and discrete optimization with uncertainty sets** (class 11)

- Interval uncertainty set, budgeted uncertainty set, ellipsoidal uncertainty set
- Adjustable robust counterpart
- Affinely adjustable robust counterpart

References:

- *A.L. Soyster. Convex programming with set-inclusive constraints and applications to inexact linear programming. Operations Research, 1973.*
- *A. Ben-Tal and A. Nemirovski. Robust solutions of uncertain linear programs. Operations Research Letters, 1999.*
- *D. Bertsimas and M. Sim. The price of robustness. Operations Research, 2004.*
- *A. Ben-Tal, A. Goryashko, E. Guslitzer and A. Nemirovski. Adjustable robust solutions of uncertain linear programs. Mathematical Programming, 2004.*
- *B. Zheng and L. Zhao. Solving two-stage robust optimization problems using a column-and-constraint generation method. Operations Research Letters, 2013.*
- *D. Bertsimas and V. Goyal. On the power of robust solutions in two-stage stochastic and adaptive optimization problems. Mathematics of Operations Research, 2010.*

(12) **Guest lecture**: Teo Chung-Piaw, NUS (class 12)

(13) **Guest lecture**: Karthyek Murthy, SUTD (class 13)

Additional References:

- *A. Ben-Tal, D. Den Hertog, A. De. Waegenaere, B. Melenberg and G. Rennen. Robust solutions of optimization problems affected by uncertain probabilities. Management Science, 2013.*
- *P. M. Esfahani and D. Kuhn. Data-driven distributionally robust optimization using the Wasserstein metric: performance guarantees and tractable reformulations. Mathematical Programming, 2018.*
- *R. Gao and A. J. Klegweyt. Distributionally robust stochastic optimization with Wasserstein distance, 2016.*
- *J. Blanchet, Y. Kang and K. Murthy. Robust Wasserstein profile inference and applications to machine learning, 2016.*
- *G. A. Hanasusanto and D. Kuhn. Conic programming reformulations of two-stage distributionally robust linear programs over Wasserstein balls. Operations Research, 2018.*