1 Supplemental Material

1.1 Application to Random Walk: Additional Numerical Results

In Figure 1, we plot the probability that the ith partial sum attains the maximum value for $i = 1, \ldots, n$ with $n = 20$. For the normal distribution, this probability is estimated through 1000000 simulations. The figure indicates that for both methods, the maximum value is attained for the extreme values $i = 1$ and $i = n$ with the highest probability while the probability of attaining the maximum reduces towards the center as $i \to n/2$ (see Mishra, Natarajan, Tao and Teo [**?**]). In Figures 2 and 3, we plot the joint probability that the ith partial sum attains the maximum value and the jth partial sum attains the minimum value simultaneously, under the extremal distribution and normal distribution respectively. The figures clearly indicate that this probability gets larger as $|i - j|$ increases and gets smaller as $|i - j|$ approaches 0.

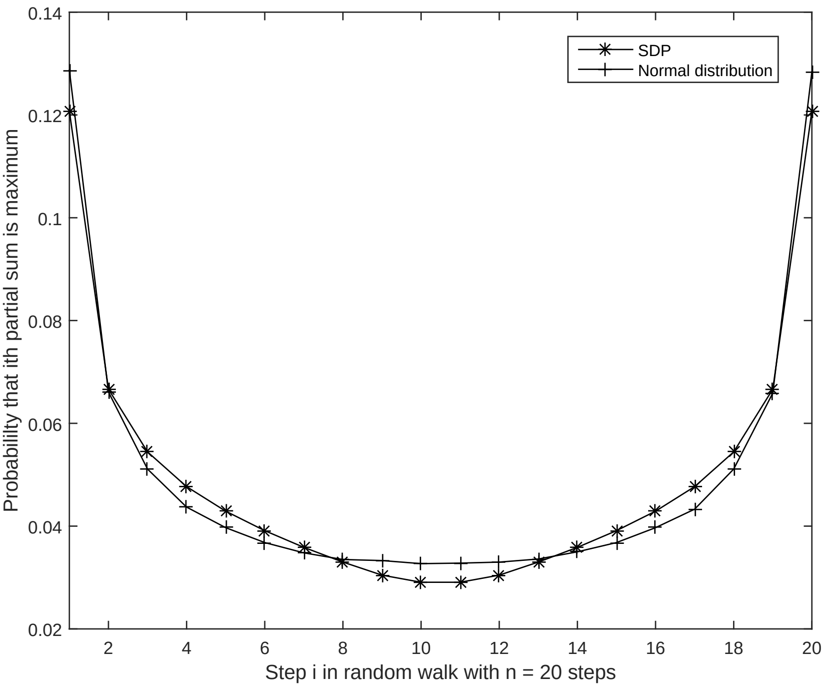

Fig. 1 Probability that the ith partial sum is the maximum.

To compare the results, we also test the scaling behavior of the SDP bound as a function of n^{α} by varying α. For $n = 1, \ldots, 60$, the scaling results are provided in Figure 4 and 5. As indicated in the figure, the SDP bound scales

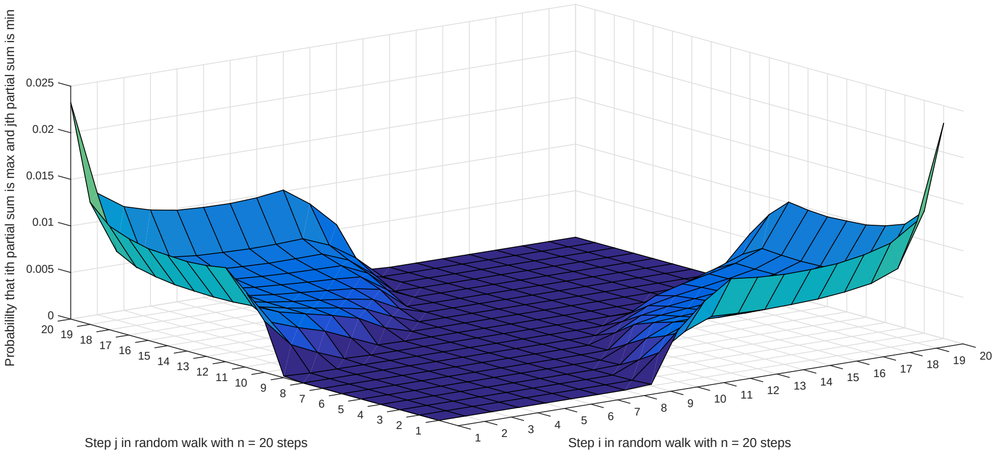

Fig. 2 Probability that the ith partial sum is the maximum and the jth partial sum is the minimum for the SDP.

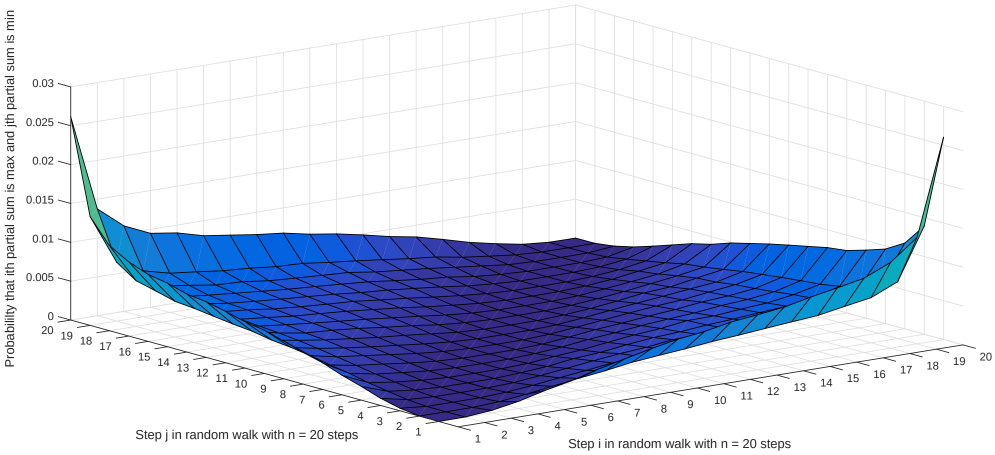

Fig. 3 Probability that the ith partial sum is the maximum and the jth partial sum is the minimum for the normal distribution.

roughly as n^α for α close to 0.7 at least for $n = 1$ to $n = 60$ which is close to Hurst's original findings. An interesting open research question is to find the asymptotic scaling behavior of the SDP bound for uncorrelated random variables with identical means and variances.

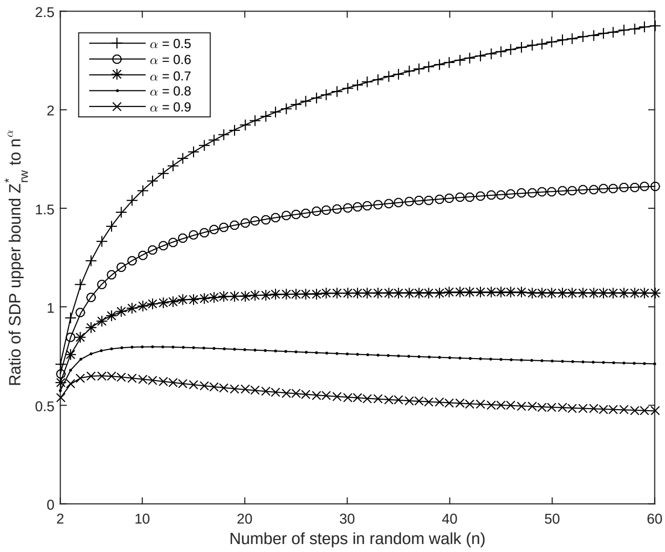

Fig. 4 Scaling of the SDP bound with n^α by varying α for $n = 2$ to $n = 60$.

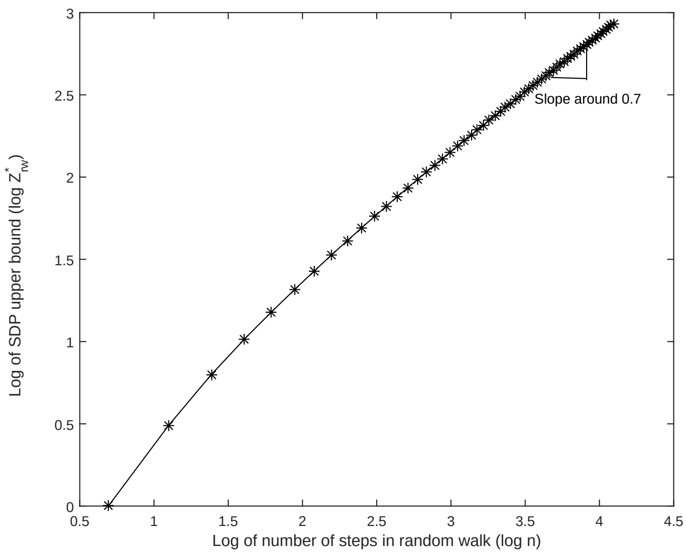

Fig. 5 Log-log graph to capture scaling of the SDP bound with n^α by varying α for $n = 2$ to $n = 60$.

1.2 Application to Best-Worst Choice Probabilities: Additional Numerical Results

In this set of numerical experiments, we assume that the error terms $\tilde{\epsilon}_i$ are uncorrelated with mean $\gamma \approx 0.5772$ and standard deviation of $\pi/\sqrt{6}$. These values for the mean and the standard deviation of the error terms are chosen so as to fit the Gumbel distribution. The systematic component of the utility v_i was randomly generated in the interval $[0, 2]$ for each i independent of each other. We estimate the best, worst and best-worst choice probabilities for $n = 10$ alternatives across 100 randomly generated sets of parameter values. In this instance, the probabilities are known in closed form for MNL. The choice probabilities for MNP were estimated through 1000000 simulations with the normal distribution while the choice probabilities for the SDP were evaluated by solving the semidefinite program. The choice probabilities are plotted in Figure 6 and provide fairly similar insights across different models.

The minimum, average and maximum total variation distance between MNL and MNP, MNL and SDP, SDP and MNP across the 100 instances are provided in Table 1.

Table 1 Total variation distance between MNL, MNP and SDP choice probabilities for $n = 10$ uncorrelated random variables.

Choice Probabilities	d(MNL,MNP)			d(MNL,SDP)			d(SDP,MNP)		
	Min	Average	Max	Min	Average	Max	Min	Average	Max
Best	0.0362	0.0616	0.0866	0.0865	0.1242	0.1578	0.0330	0.0650	0.0829
Worst	0.0655	0.1128	0.1570	0.0220	0.0668	0.1139	0.0319	0.0660	0.0898
Best-Worst	0.0876	0.1285	0.1680	0.1557	0.2148	0.2659	0.1298	0.2041	0.2685

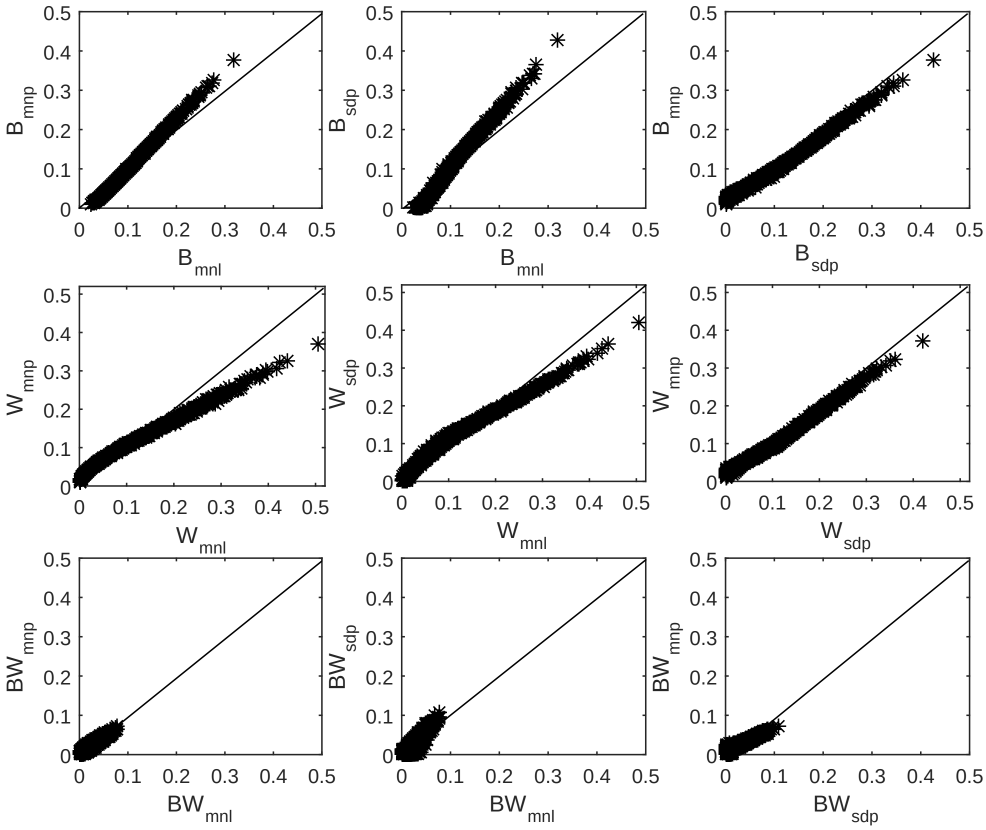

Fig. 6 Scatter plots of best, worst and best-worst choice probabilities using MNL, MNP and SDP for $n = 10$ uncorrelated random variables.