Dynamic Pricing for Controlling Age of Information (AoI)

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ESD ENGINEERING SYSTEMS & DESIGN

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Key Research Questions

- To keep the freshness of content updates, many content providers (e.g., Waze and Crowdspark) now invite and pay the mobile users (with sensors embedded in their portable devices) to sense and send back real-time useful information.
- When motivating crowd to contribute real-time information, how to trade off between sampling cost and AoI reduction?
- Mobile users arrive from time to time with uncertainty and have private sampling costs unknown to the content providers.
- How should a content provider decide dynamic pricing to balance the monetary payments to users and the AoI evolution over time under incomplete information about users' arrivals and their private sampling costs?

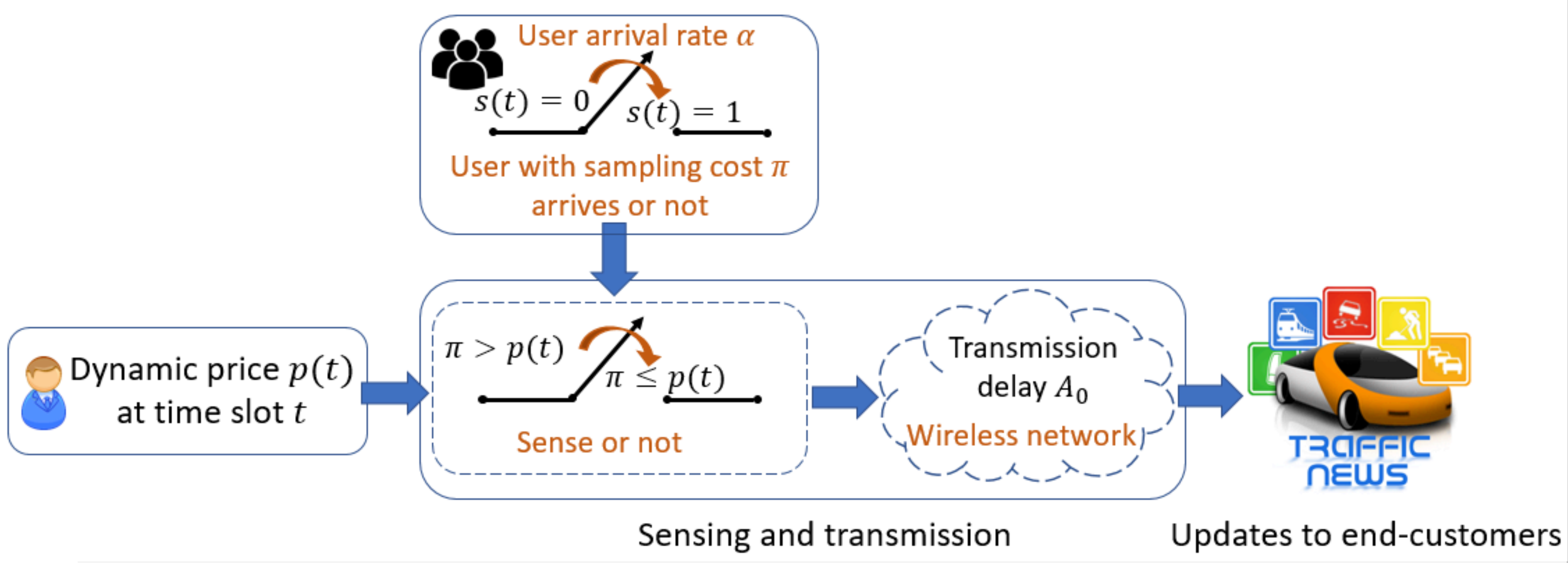
System model

Many content providers such as Waze and CrowdSpark now invite and pay the mobile crowd including smartphone users and drivers to sample real-time information frequently.

This translates to high sampling cost when inviting many sensors to sense and update, and should be also taken into account when managing the AoI.

As AoI $A(t)$ changes over time, the compensation price $p(t)$ at time t should be dynamic and age-dependent to best balance the AoI evolution and the sampling cost.

Yet this dynamic programming problem is challenging to solve especially under the incomplete information about users' arrivals ($s(t)$=0 or 1) and sampling cost π in support $[0, b]$ and CDF $F()$.



1) The provider first announces dynamic price $p(t)$ at the beginning of each time slot t,
2) a user may arrive randomly in this time slot and (if so or $s(t)=1$) he further decides to sample or not based on the price and its own sampling cost.
3) If the user appears and accepts to sample with $\pi > p(t)$, its sensor data is transmitted with fixed delay A_0 to finally post in the provider's platform.

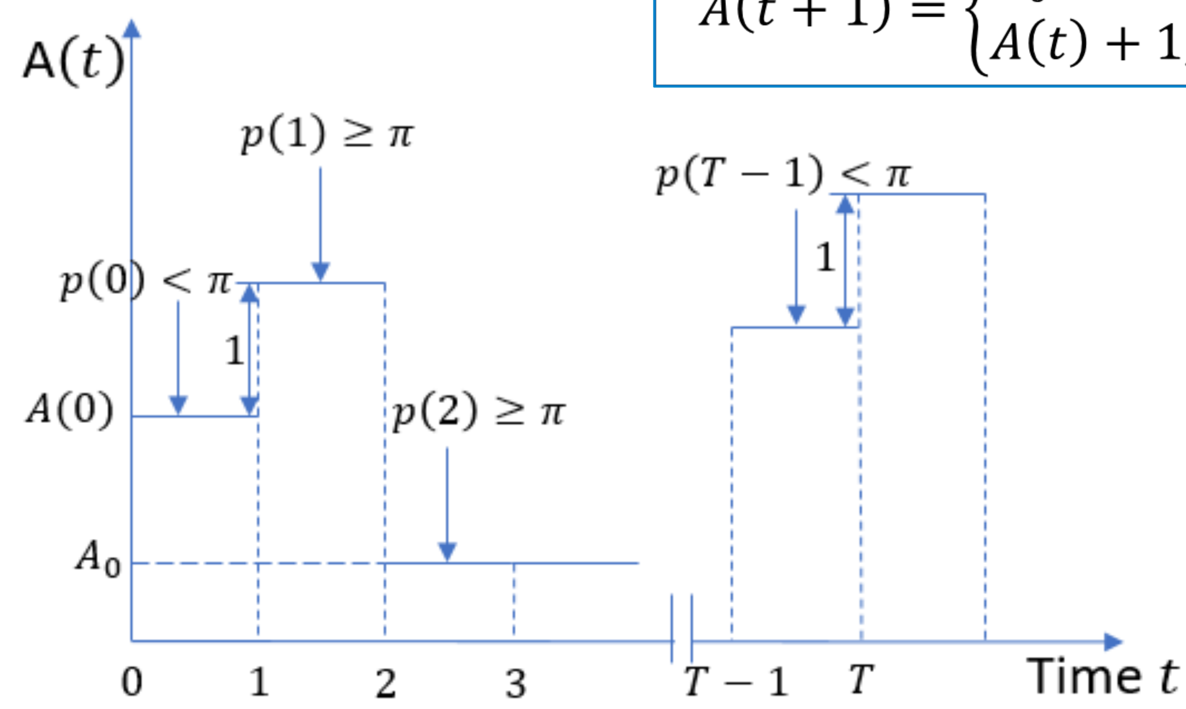
Research Output

X. Wang & L. Duan, "Dynamic Pricing for Controlling Age of Information," to appear in *IEEE ISIT* 2019. https://arxiv.org/abs/1904.01185

Key Approach and Results

AoI evolution over time under dynamic pricing in a discrete-time horizon:

$$A(t+1) = \begin{cases} A_0, & \text{if } \pi \le p(t) \ \& \ s(t)=1; \\ A(t)+1, & \text{otherwise.} \end{cases}$$



Dynamics of the expected AoI:

$$\begin{aligned} A(t+1) &= \alpha F(p(t))A_0 + \left(1-\alpha F(p(t))\right)(A(t)+1) \\ &= A(t) - (A(t)-A_0)\frac{\alpha p(t)}{b} + \left(1-\frac{\alpha p(t)}{b}\right). \end{aligned} \quad (1)$$

The objective of the provider is to find the optimal dynamic pricing $p(t) \in [0,b], t \in [0,\ldots,T]$ to minimize the expected discounted cost :

$$U(T) = \min_{p(t)\in[0,b],t\in[0,\ldots,T]} \sum_{t=0}^{T} \rho^t \left(A^2(t) + \frac{\alpha p^2(t)}{b}\right).$$

Tradeoff between AoI $A^2(t)$ and expected payment $\frac{\alpha p^2(t)}{b}$

Constrained nonlinear dynamics, challenging to solve analytically due to the curse of dimensionality (computation complexity $O((\frac{b}{\varepsilon})^T)$). To analytically solve the optimal price, we linearize the nonlinear AoI evolution in (1), by approximating the dynamic AoI reduction $A(t) - A_0$ as a time-average term $\delta = \frac{1-\rho}{1-\rho^T}\sum_{t=0}^{T-1}\rho^t(A(t)-A_0)$:

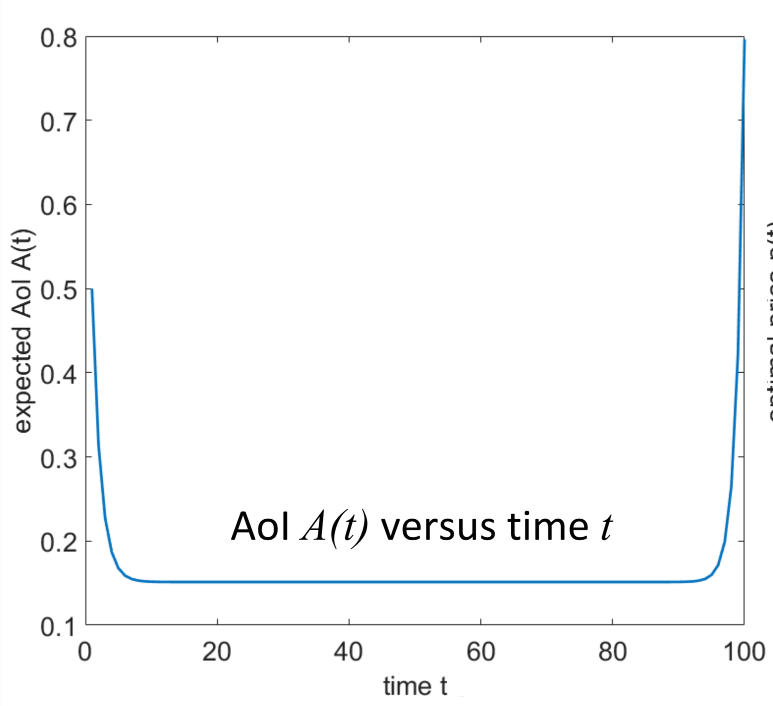
$$A(t+1) = A(t) - \delta\frac{\alpha p(t)}{b} + \left(1-\frac{\alpha p(t)}{b}\right).$$

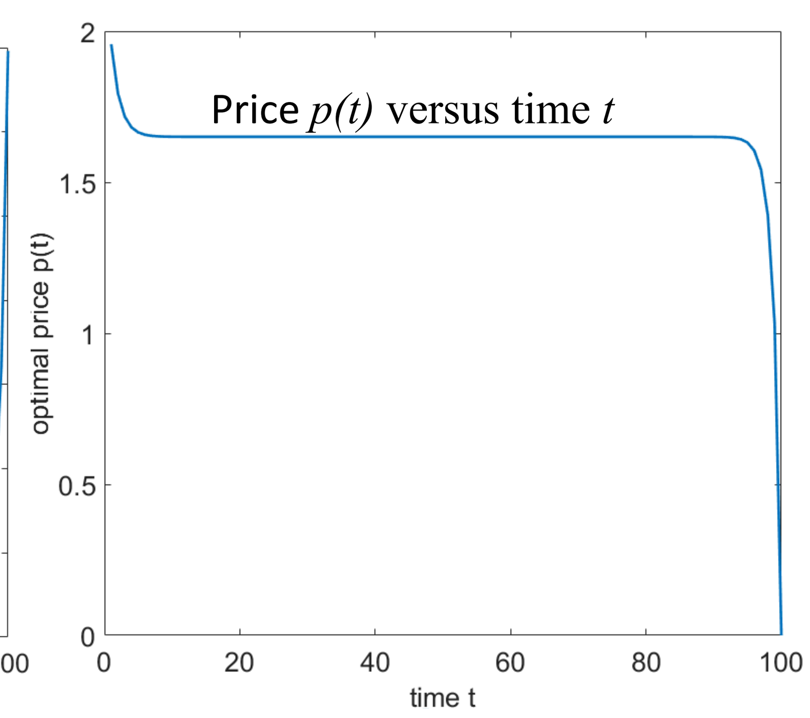
Fixed point δ exists.

The optimal approximate dynamic pricing $p(t), t \in [0,\ldots,T]$ increases in $A(t)$:

$$p(t) = \frac{\rho M_{t+1}(\delta+1) + 2\rho(\delta+1)Q_{t+1}(A(t)+1)}{2 + \frac{2\rho Q_{t+1}\alpha(\delta+1)^2}{b}},$$

with $p(T)=0, Q_T=1, M_T=0$. $Q_t = 1 + \frac{\rho Q_{t+1}}{1+\frac{\rho Q_{t+1}\alpha(\delta+1)^2}{b}}$, $M_t = \frac{\rho(M_{t+1}+2Q_{t+1})}{1+\frac{\rho Q_{t+1}\alpha(\delta+1)^2}{b}}$,





Dynamic pricing $p(t)$ first increases with AoI $A(t)$ until both of them reach steady-states.

But when close to the end of the time horizon, price decreases to $p(t=T)=0$ to save sampling expense, w/o worrying future AoI.

If we use the steady-state price $p^\infty(t)$ without iteratively computing Q_t and M_t, the resulting expected discounted cost $U^\infty(T)$ for finite horizon T *approaches the optimal* expected discounted cost $U(T)$ as $T \to \infty$.

$$p^\infty(t) = \frac{\rho M(\delta+1) + 2\rho(\delta+1)Q(A(t)+1)}{2+\frac{2\rho Q\alpha(\delta+1)^2}{b}}.$$