Economics of UAV-provided Mobile Services

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ABSTRACT
Due to its agility and mobility, the unmanned aerial vehicle (UAV) is a promising technology to rapidly provide mobile services (e.g., wireless coverage, edge computing, and local caching) to users. Yet the UAV has limited energy storage and needs to fly to serve users locally, requiring an optimal energy allocation for balancing both hovering time and service capacity. For profit-maximizing, given hovering in a hotspot, how the UAV should dynamically price its capacity-limited service according to randomly arriving users with private service valuations is another concern. This paper first analyzes the UAV’s optimal pricing under incomplete user information in a hotspot, and also optimizes the energy allocation to hovering time and service capacity. We prove that the UAV should ask for a higher price if the leftover hovering time is longer or its service capacity is smaller, and its expected profit approaches to that under complete user information if the hovering time is sufficiently large. As the hotspot’s mean user density increases, a shorter hovering time or a larger service capacity should be allocated.

Finally, facing multiple hotspot candidates with different user densities and flying distances in the ground plane, the UAV still needs to decide its deployment and we prove that it is optimal to deploy the UAV to serve a single hotspot. With multiple UAVs, this result can be reversed with UAVs’ forking deployment to different hotspots when hotspots are more symmetric or the UAV number is large.

1 INTRODUCTION
Due to the fast deployment and controllable mobility, the unmanned aerial vehicle (UAV) emerges as a promising technology to rapidly provide mobile services (e.g., wireless coverage, edge computing, and local caching) to ground users. For example, Verizon has developed a drone-based cell service system to provide 4G LTE cell service for the area out of service [10]. By endowing with edge computing capabilities, the UAV can be also used to offer computation offloading services to mobile users with limited local processing capabilities [4]. Moreover, cache-enabled UAV is implemented to improve the quality-of-experience (QoE) of mobile devices by caching and distributing the popular content to them [1]. The global profit of such UAV-enabled services is expected to increase from $792 million in 2017 to $12.6 billion by 2025 [8], and more service providers are seeing the opportunities to employ UAVs and make a profit.
This paper proposes a three-stage decision making model to analyze the UAVs’ deployment, energy allocation and dynamic pricing. At Stage I, given a number $N$ of identical UAVs at the UAV station, the provider of UAV-provided services should decide the optimal deployment of the UAVs to $M$ potential hotspots as shown in Fig. 1 under the UAV’s total energy budget $B_0$. Each hotspot $m$’s user density and flying distance from UAV station are denoted as $\alpha_m$ and $D_m$, $m = 1, ..., M$, respectively. More precisely, $\alpha_m$ tells the UAV’s probability of meeting a user’s service request in every time slot due to the user’s random mobility.

At Stage II, given a UAV’s energy storage $B$ upon arrival at a hotspot, it should decide the energy allocation to the hovering time $T$ and service capacity $k$ with $T + c k \leq B$, where $c > 0$ is the energy consumption for serving a user (relative to the energy consumption per hovering time). If the UAV hovers longer, it may encounter more users for higher pricing under incomplete information, yet the number $k$ of users it can serve decreases given the energy budget $B$.

At Stage III, given the hovering time $T$ and service capacity $k$, the UAV determines the dynamic pricing strategy $\{p_1(t), ..., p_k(t)|t = 1, ..., T\}$ at any time $t$ for any leftover capacity as shown in Fig. 1 based on the distribution of the users’ service valuations $v$ and user arrival process, where $p_j(t), j = 1, ..., k$, is the price for selling the service to the $j$th-to-last user at $t$ time units before the end of selling interval $T$. Note that $t = 0$ ($t = T$) means the end (beginning) of the service interval and $p_j(t)$ $(p_k(t))$ mean the price for serving the last (first) user. For simplicity, we will use “time units before the end of serving interval $T$” and “time $t$ interexchangeably in the rest of the paper. A user will accept the price if his service valuation $v$ is no less than the price asked by the provider. It is assumed that there is no possibility of recall the users who already leave. For the incomplete information case considered in this paper, the provider does not know the user request appearance over time $t$ or the user’s private service valuation $v$. It only knows the appearance probability in each time slot and the distribution of user’s i.i.d. service valuation $\{v|v \in [a, b]\}$.

In the following, we use backward induction to analyze the UAV’s decision at each stage. Due to the page limitation, most of the proofs are given in the technical report [9].

3 UAV’S DYNAMIC PRICING

At Stage III, a UAV’s hovering time $T$ and service capacity $k$ are given for a particular hotspot with the probability $\alpha$ of meeting a user service request. Here we skip the subscript as the analysis holds for a typical UAV at any hotspot. The UAV should decide the dynamic pricing $p_j(t)$ at $t$ time slots before the end of hovering interval $T$ given any leftover service capacity $j, j = 1, ..., k$ as shown in Fig. 1 without knowing user request arrival in the future.

Before studying the UAV’s optimal pricing strategy for any service capacity $k$, we first consider the case of $k = 1$. By announcing price $p_1(t)$ at time $t$, a user (if appears with probability $\alpha$) will accept and pay the price if his service valuation $v$ is greater, i.e., $v \geq p_1(t)$. Given the cumulative distribution of his service valuations $F(v)$, the probability that a user will appear and accept the price is $\alpha(1 - F(p_1(t)))$. Then, the expected total profit in the remaining $t$ time slots is

$$R_t(t) = \alpha p_1(t)(1 - F(p_1(t))) + R_t(t - 1)(1 - \alpha(1 - F(p_1(t)))) \quad (1)$$

We then consider the case of $k = 2$. After successfully serving the first user, the profit analysis of the case $k = 1$ in
(1) can be applied for subsequently serving the second user. Note that the expected total profit received from the second user at \( t \) time slots is \( R_1(t) \). By successfully serving the first user at price \( p_2(t) \) at time \( t \), the UAV’s total profit at time \( t \) is \( p_2(t) + R_1(t-1) \). Then, the expected profit with service capacity \( k = 2 \) at time slot \( t \) is

\[
R_2(t) = \alpha (p_2(t) + R_1(t-1))(1 - F(p_2(t))) + R_2(t-1)(1 - \alpha (1 - F(p_2(t)))) \tag{2}
\]

Similar to the above analysis, for the general \( k \geq 2 \) case, the expected total profit at time \( t \) can be derived recursively as

\[
R_k(t) = \alpha (p_k(t) + R_{k-1}(t-1))(1 - F(p_k(t))) + R_k(t-1)(1 - \alpha (1 - F(p_k(t)))), \tag{3}
\]

where \( R_k(0) = 0 \) with zero remaining hovering time and \( R_0(t) = 0 \) with zero leftover service capacity.

By taking the derivative of \( R_k(t) \) with respect \( p_k(t) \), the optimal price \( p_k(t) \) satisfies

\[
\frac{dR_k(t)}{dp_k(t)} = \alpha (1 - F(p_k(t)) - f(p_k(t)) \times (p_k(t) - (R_k(t-1) - R_{k-1}(t-1)))) = 0 \tag{4}
\]

**Proposition 3.1.** For \( \forall t \in \{1, \cdots , T\} \) and \( k \geq 1 \), if \( t < k \), \( R_k(t) = R_t(t) \) and the price \( p_k(t) \) at time \( t < k \) does not exist. If the UAV has enough leftover time to sell out \( k \) (i.e., \( t \geq k \)), it is better to optimize its expected profit \( R_k(t) \) via jointly pricing \( k \) service capacities over \( t \) time slots, rather than independently pricing each service capacity with \( t \) hovering time.

In the following, we assume the distributions of the service valuations are regular, which is widely used in the realm of mechanism design [2]; \( \phi(v) = v - \frac{1 - F(v)}{F(v)} \) is an increasing function of \( v \), where \( F(v) \) and \( f(v) \) are the CDF and PDF of the user’s valuation \( v \). Under this assumption, the solution \( p_k(t) \) to (4) is the unique optimal price.

**Proposition 3.2.** For \( \forall t \in \{1, \cdots , T\} \) and \( k \geq 1 \), Algorithm 1 optimally computes the dynamic pricing scheme with computation complexity \( O(KT) \). Especially, when \( k = 1 \), the optimal price \( p_1(t) \) is a non-decreasing function of leftover time \( t \) and mean user density \( \alpha \) in the hotspot.

In Algorithm 1, we computes the optimal price \( p_j(t) \) according to (4) with initial conditions \( R_1(0) = 0 \) and \( R_0(t) = 0 \). Proposition 3.2 also shows that the UAV should ask for a higher price if it has more leftover time \( t \) for encountering more users or a greater user demand.

### 3.1 Continuous-time Relaxation

In the discrete time model as in Fig. 1, we can only use a recursive and numerical way to derive \( R_k(t) \) and \( p_k(t) \) according to (3). To obtain more analytical results for dynamic pricing design, we next apply continuous-time relaxation on the discrete time model. Assume users arrive according to a Poisson process with arrival rate \( \alpha' \). Denote the time duration of each time slot for discrete time model as \( \varepsilon \). To keep the same user arrival probability \( \alpha \) for discrete time case, we have \( \alpha' + o(\varepsilon) = \alpha \) as \( \varepsilon \to 0 \). Similar to the analysis of discrete time case in (3), as \( \varepsilon \to 0 \), the expected total profit that a provider can obtain at time \( t + \varepsilon \) is

\[
R_k(t + \varepsilon) = \alpha' \int_{t}^{t+\varepsilon} R_k(x) + R_{k-1}(x)(1 - F(p_k(x)))dx + R_k(t)(1 - \alpha' \int_{t}^{t+\varepsilon} (1 - F(p_k(x)))dx + o(\varepsilon)). \tag{5}
\]

Note that \( R_k(0) = 0 \), according to (5), the expected total profit with service capacity \( k \) at time \( t \) can be derived as

\[
R_k(t) = \alpha' \int_{0}^{t} \{ (p_k(x) + R_{k-1}(x) - R_k(x))(1 - F(p_k(x)))dx \}. \tag{6}
\]

Thus, the optimal price \( p_k(t) \) that maximizes the expected profit \( R_k(t) \) is

\[
p_k(t) = \arg \max_{p \geq R_k(t) - R_{k-1}(t)} (p + R_{k-1}(t) - R_k(t))(1 - F(p)). \tag{7}
\]

To analytically obtain the expected profit, we further consider the case that the users’ i.i.d. service valuations follow exponential distributions, i.e., \( F(v) = 1 - e^{-\lambda v} \). Then, by solving (7), the optimal price \( p_k(t) \) is

\[
p_k(t) = \frac{1}{\lambda} + R_k(t) - R_{k-1}(t), \tag{8}
\]

which is greater than the mean valuation \( \frac{1}{\lambda} \) by considering the future pricing opportunity. Insert (8) into (6), the expected profit for serving \( k \) users in hovering time \( T \) can be
derived as

\[ R_k(T) = \frac{1}{\lambda} \log \left( \sum_{i=0}^{k} \frac{1}{i!} \left( \frac{\alpha t^i}{e} \right) \right). \]

(9)

And the dynamic price in (8) at time \( t \in [0, T] \) is

\[ p_k(t) = 1 + \frac{1}{\lambda} \log \left( \sum_{i=0}^{k} \frac{1}{i!} \left( \frac{\alpha t^i}{e} \right) \right). \]

(10)

**Proposition 3.3.** The optimal expected profit \( R_k(T) \) concavely increases with both \( k \) and \( T \), respectively. Further, the optimal price \( p_k(t) \) increases with \( t \) and convexly decreases with \( k \).

Proposition 3.3 shows that the growth rate of the expected profit decreases with service capacity \( k \) given the fixed hovering time \( T \). This is because the partitioned hovering time \( \frac{T}{k} \) for pricing and serving one user decreases with \( k \). Similarly, the growth rate of the expected profit decreases with hovering time \( T \) given the fixed service capacity \( k \). Finally, the optimal price increases faster as \( k \) decreases.

**Comparison with complete information case:** As the dynamic pricing under incomplete information is analyzed above, we wonder the performance gap between the incomplete information and complete information, where the UAV can perfectly observe a user’s service valuation \( v \) upon arrival. According to the threshold-based assignment policies discussed in [3], we have the following proposition.

**Proposition 3.4.** For \( k \in \{1, 2, 3\} \), \( \lim_{T \to \infty} \frac{R_k(T)}{R_k(T)} = 1 \), where \( R_k(T) \) and \( \hat{R}_k(T) \) are the expected profits under incomplete and complete information for exponential distribution of users service valuations, respectively.

Actually, for any finite \( k < \infty \), we can iteratively obtain \( \hat{R}_k(T) \) according to [3] and show the convergence of \( \frac{R_k(T)}{\hat{R}_k(T)} \) to 1. As shown in Fig. 2, \( R_k(t) \) approaches \( \hat{R}_k(t) \) if the hovering time \( T \) is sufficiently large. Moreover, \( R_k(t) \) converges faster to \( \hat{R}_k(t) \) as the service capacity \( k \) decreases. This is because, as \( k \) decreases, the partitioned hovering time \( \lfloor \frac{T}{k} \rfloor \) for pricing and serving one user increases given the hovering time \( T \).

## 4 UAV’S ENERGY ALLOCATION IN HOVERING TIME AND SERVICE CAPACITY

As a longer hovering time \( T \) results in a higher service price at the cost of smaller service capacity \( k \), at Stage II the UAV under the total energy budget \( B \) should balance \( T \) and \( k \). Its optimal energy allocation problem at the given hotspot is

\[
\max_{k, T \in \mathbb{Z}^+} R_k(T),
\]

s.t.

\[
T + ck \leq B,
\]

where \( B \) can be viewed as the maximum hovering time if the UAV does not use any energy to serve any user.

At the optimality, (12) is tight to use up all budget and the problem can be rewritten as

\[
\max_{k \in \mathbb{Z}^+} R_k(B - ck),
\]

(13)

Recall that \( \forall t < k, R_k(t) = R_k(t) \) in Proposition 3.1 and the UAV would not set \( k \) to be larger than the maximum time, i.e., \( k \leq B - ck \). Thus, integer \( k \) is upper bounded by \( \lfloor \frac{T}{1 + \alpha} \rfloor \). By using Algorithm 1 for any \( k \in \{1, \ldots, \lfloor \frac{T}{1 + \alpha} \rfloor \} \), we can calculate the corresponding expected profit \( \hat{R}_k(B - ck) \).

Then, the UAV compares and chooses the \( k^* \) with maximal expected profit, i.e., \( k^* = \arg \max_k \hat{R}_k(B - ck) \).

Fig. 3 shows a numerical example for uniform distribution of service valuations.

- For low density \( \alpha \), it is better to only serve one user, i.e., \( k^* = 1 \). It is worthwhile for the UAV to hover the longest time to encounter a user.
- For high density \( \alpha \), it is easier for the UAV to encounter more users and it should choose \( k^* \in \{2, \ldots, \lfloor \frac{T}{1 + \alpha} \rfloor \} \), telling an optimal balance between encountered demand and service capacity supply.

Besides the numerical result, we also want to obtain analytical results for the energy allocation policy. Similar to
Section 3.1, we apply continuous-time relaxation, where the maximum service capacity is $\frac{Q}{M}$. In the following theorem, we successfully derive a threshold-based energy allocation policy assuming that the users’ service valuations follow exponential distributions.

**Theorem 4.1.** The optimal service capacity $k^*$ depends on the user density and is given as follows.

- **Low user density regime** ($\alpha' \leq \frac{2e}{(2-2e)T}$): the UAV will decide $k^* = 1$ for serving one user only.
- **High user density regime** ($\alpha' > \frac{2e}{(2-2e)T}$): the UAV will decide

$$k^* = \arg \max_{k \in \mathbb{Z}^+} \sum_{i=0}^{k-1} \frac{\alpha'(B - ck)}{e} \in \{2, \cdots, \left\lceil \frac{B}{c} \right\rceil \}.$$  \hspace{1cm} (14)

The analytical result in Theorem 4.1 is consistent with Fig. 3 for uniform distributions under discrete time case.

**5 OPTIMAL UAV DEPLOYMENT**

Given $M$ hotspots with heterogeneous user densities $\alpha_m$ and distances $D_m$ from the UAV station with $m \in \{1, \ldots, M\}$, we now study how to deploy $N$ identical UAVs each with an energy budget $B_0$ to these hotspots.

**5.1 Deployment of a Single UAV**

Given a single UAV’s route going through $M' \leq M$ hotspots in sequence $\mathcal{H} = \{H_1, H_2, \ldots, H_{M'}\}$, the route distance is $D_{H_1} + \sum_{m=1}^{M'-1} D_{H_m, H_{m+1}}$, where $D_{H_1}$ and $D_{H_m, H_{m+1}}$ are the flying distances from the UAV’s station at the original point to hotspot 1 and from hotspot $H_m$ to hotspot $H_{m+1}$, respectively. Given the initial energy budget $B_0$, the UAV partitions remaining energy to $M'$ hotspots as $B_{M'} = \{B_{H_1}, \ldots, B_{H_{M'}}\}$ with $\sum_{m=1}^{M'} B_{H_m} = B_0 - D_{H_1} - \sum_{m=1}^{M'-1} D_{H_m, H_{m+1}}$. Given energy budget $B_{H_m}$ for hotspot $H_m$, we still need to decide the energy allocation to hovering capacity $T_{H_m}$ and service capacity $k_m$ as well as the dynamic pricing during hovering time $T_{H_m}$. Based on (13) and Algorithm 1, we can obtain the overall expected profit for the whole route problem as

$$\Psi_\mathcal{H}(B_{M'}) = \sum_{H_m \in \mathcal{H}} \max_{k_m \in \mathbb{Z}^+} R_{k_m H_m}^0 (B_{H_m} - ck_{H_m}),$$  \hspace{1cm} (15)

where $R_{k_m H_m}^0 (t)$ is the expected profit of hotspot $H_m$ with user density $\alpha_{H_m}$ for serving $k_{H_m}$ users at time $t$.

Still, the UAV needs to decide the route by considering $\sum_{M'=1}^{M} C_M^{M'}$ possible routes, where the UAV needs to choose $M'$ out of $M$ hotspots and each $M'$ introduces $M'$ possible sequences. Among these routing possibilities, the UAV needs to find the optimal route with corresponding hotspot sequence $\mathcal{H}$ and the corresponding optimal energy allocation $B_{M'}$ to maximize the overall expected profit

$$\max_{\mathcal{H}} \max_{B_{M'}} \Psi_\mathcal{H}(B_{M'}).$$  \hspace{1cm} (16)

For analysis tractability, we apply continuous-time relaxation and analyze the optimal UAV routing assuming exponential distributions of users’ service valuations.

**Theorem 5.1.** For any number $M$ of hotspots distributed in the ground plane, it is optimal to deploy the UAV to only one hotspot $m^* = \arg \max_{m \in \{1, \ldots, M\}} R_{k_m}^0 (B_0 - D_m - ck_m)$.

Theorem 5.1 tells that the single UAV will only serve the first best hotspot with maximum expected profit. If part of energy budget is removed from the first best hotspot to also serve the second best, the UAV’s marginal profit from serving the second best hotspot is lower. Thus, it is not worthwhile to deploy the UAV to more than one hotspot.

**5.2 Deployment of Multiple UAVs to Hotspots: Forking or Not**

We are now ready to study how to assign multiple UAVs from a common UAV station to heterogeneous hotspots, and we focus on the key question: whether all the UAVs should still be deployed to the first best hotspot or they will fork to serve different hotspots. It is possible for more than one UAV to jointly serve the same hotspot by grouping their service capacities. Given a number $n_m$, of UAVs jointly serving hotspot $m$, $m \in \{1, \ldots, M\}$, they will stay for the same amount of time $T_m$ due to their symmetry. To keep the same energy consumption rates, $n_m$ UAVs take turns to serve a user’s request. For example, in the caching application, each UAV can sequentially distribute $1/n_m$ segment of a popular file to a user in request. For a particular hotspot $m$, the objective of the UAVs is to maximize their total expected profit

$$\max_{K_m \in \mathbb{Z}^+} R_{k_m}^0 (T_m),$$  \hspace{1cm} (17)

s.t. $n_m T_m + ck_m \leq n_m (B_0 - D_m)$,  \hspace{1cm} (18)

where they decide total service capacity $k_m$ by aggregating their residual energy $n_m (B_0 - D_m)$.

Then, the overall expected profit under the UAV assignment $N = \{n_1, \ldots, n_M\}$ for $M$ hotspots is

$$\Phi(N) = \sum_{m=1}^{M} \max_{k_m \in \mathbb{Z}^+} R_{k_m}^0 \left( [B_0 - D_m - ck_m] \right).$$  \hspace{1cm} (19)

Note that $R_{k_m}^0 = 0$ for hotspot $m$ if $n_m = 0$. For the discrete time model, the service capacity $k_m$ at hotspot $m$ should be no larger than the maximum time, i.e., $k_m \leq B_0 - D_m - \frac{ck_m}{n_m}$. Thus, $k_m \leq \left\lfloor \frac{B_0 - D_m}{1/n_m + c} \right\rfloor$. Given any UAV deployment $N$, we can obtain the optimal expected profit

$$\max_{N} \Phi(N) \leq \Phi(\hat{N}) \leq \max_{N \in \mathcal{A}(\hat{N})} \Phi(N).$$

Without loss of generality, we sort the $M$ hotspots according to their expected profits served from a single UAV, i.e.,

$$R_{k_m}^0 (B_0 - D_m - ck_m) \geq \cdots \geq R_{k_{M'}}^0 (B_0 - D_m - ck_{M'}).$$  \hspace{1cm} (20)
As the user densities of hotspots where \( k_m = \arg \max_{k \in \mathbb{Z}^+} R^m_{k_m}(B_0 - D_m - ck_m), \ m \in \{1, \ldots, M\} \). In the following proposition, we show the sufficient condition whether UAVs will all center at the first best hotspot 1 or fork to hotspot 2 (or more) assuming exponential distributions of users’ service valuations for continuous-time scenario.

**Proposition 5.1.** Given \( N \geq 2 \) UAVs for \( M \geq 2 \) hotspots, the UAVs will fork in their deployment to serve different hotspots if

\[
a'_2 > a'_1 \max(\varphi^{\frac{1}{2}}, \varphi),
\]

where \( \varphi \) is an increasing function of \( D_2 \) given as

\[
\varphi = \frac{\max_{k_1} \sum_{i=0}^{k_1} \frac{1}{i!} \left( \frac{a'_1(B_0 - D_1 - \frac{c D_2}{N})}{e} \right)^i (\sum_{i=1}^{k_2} \frac{1}{i!} \left( \frac{a'_1(B_0 - D_2 - \frac{c D_2}{N})}{e} \right)^i)}{\sum_{i=1}^{k_2} \frac{1}{i!} \left( \frac{a'_1(B_0 - D_2 - \frac{c D_2}{N})}{e} \right)^i}.
\]

According to Proposition 5.1, we can see that the UAVs are more likely to fork to serve different hotspots when the hotspots are more symmetric. We also wonder the impacts of the number of UAVs on the deployment decision of forking. As an numerical example, we consider 5 UAVs to be deployed as shown in Figs. 4(a) and 4(b). Figure 4: Illustration of the optimal UAV assignment of multiple UAVs in different hotspot networks when \( B_0 = 20, c = 2 \), where the number in red circle indicates how many UAVs are assigned to the corresponding hotspot.

\[
\text{(a) UAV deployment } N = \{3, 2, 0, 0, 0\} \quad \text{(b) UAV deployment } N = \{5, 0, 0, 0, 0\} \quad \text{(c) UAV deployment } N = \{5, 2, 1, 1, 0\}.
\]

6 CONCLUSION

In this paper, we first analyze the UAV’s dynamic pricing under incomplete information including random user arrivals and unknown service valuations. It is shown that the UAV should ask for a higher price if the leftover hovering time is longer or its service capacity is smaller, and its expected profit approaches to that under complete user information if the hovering time is sufficiently large. Then, given a hotspot, the energy allocation to hovering time and service capacity is optimized. We show that, given the energy budget at the hotspot, a shorter hovering time or a larger service capacity should be allocated as the hotspot’s mean user density increases. For the UAV deployment, we prove that it is optimal for a single UAV to only serve the best hotspot. While for multiple UAVs, they prefer to fork to serve different hotspots when hotspots are more symmetric or the UAV number is large.

REFERENCES


