User-initiated Data Plan Trading via A Personal Hotspot Market

Xuehe Wang, Lingjie Duan, and Rui Zhang

Abstract

Mobile data services are becoming the main driver of a wireless service provider's (WSP's) revenue growth, and two-part tariff data plans (each including a lump-sum fee and a per-unit charge) are usually provided to wireless users. Some users can easily use up their monthly data quota and may pay for costly data over-usage. Motivated by users' diverse usage behavior (more or less than the subscribed data quotas), this paper proposes a new type of user-initiated network for cellular users to trade data plans by leveraging personal hotspots (PHs) with users' smartphones. A user with data surplus can set up a PH and share the cellular data connection to another user with data deficit in the vicinity. Due to users' randomness in data usage, incentive to trade, and user mobility to enter or leave the PH connection range, the analysis on the user-initiated network is challenging. To overcome these issues, we propose a PH-market for users with diverse data usage behaviors and random user mobility to directly trade data as sellers and buyers, by designing a market-clearing price. It is shown that the PH-market greatly saves all users' expected costs when the existence condition of the PH-market is met. Finally, as this PH-market will challenge the WSP's revenue collection (especially the surcharge from users' data over-usage), we analyze the WSP's response to the PH-market and propose two effective countermeasure strategies by either reducing the selling users' data quota in their data plans (the PH-market's supply) or increasing the buying users' data quota (the PH-market's demand). When we have more than one WSPs and they are competitive, we show one WSP can take advantage from the PH-market by indirectly selling more data to the other WSP's users.

Index Terms

Two-part tariff data plans, personal hotspot, network economics, market-clearing price

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I. Introduction

With the rapid developments of mobile Internet technologies and social services, the mobile data services are becoming the main driver of revenue growth of a wireless service provider (WSP) [1]. Most WSPs (e.g., AT&T and China Mobile) in the world are providing two-part tariff data plans to wireless users. After subscribing to a two-part tariff data plan denoted by (B, P, p), a user is given a monthly data cap B at a fixed lump-sum fee P and should pay for extra data beyond B at a costly unit price p. Due to the diversity of users' data usage behaviors, some users have leftover data, while some others face data deficit and need to pay costly extra fee to the WSP. Thus, it is beneficial for users to potentially trade monthly data themselves and save their total costs.

With the technological advancements in smartphones, many iPhones and Android phones can now set up personal hotspots (PHs) to share the cellular data connections to nearby wireless devices [2]. The physical coverage of a hotspot ranges hundreds of feet like WiFi and is expected to further increase in the future [3]. However, the development of such user-initiated PH networks is still lack of a clear business model and a user only shares the PH connection with his own or friends' devices. Now we provide an illustrative example to tell the users' economic benefits by leveraging PHs for data plan trading. User 1 and user 2 stay in the same workplace most of the time and they subscribe to the same two-part tariff data plan (B, P, p)=(3 GB, CNY 100, CNY 0.3 per MB) provided by China Mobile, where CNY represents "Chinese Yuan or Renminbi (RMB)". Assume users 1 and 2 can precisely predict their monthly usages to be 2 GB and 3.8 GB before trading, respectively. Their original costs are CNY 100 without any surcharge and CNY 340 with CNY 240 surcharge, respectively. By selling 0.8 GB as a part of the unused data from user 1 to 2 at a reasonable price CNY 0.2 per MB (lower than p), user 2's cost is reduced to CNY 260 and user 1 even gains revenue, i.e., CNY 60, after paying to the WSP. As user 2's over-usage cost paid to the WSP becomes 0 after trading data, the WSP's revenue from these two users is reduced from CNY 440 to CNY 200. In practice, a user's data usage is random and he cannot predict his future data demand exactly before trading. Under such uncertainty, we are interested to find who the seller or buyer is and how much data to trade.

In this paper, we propose a novel PH-market for demand-heterogeneous users to trade their monthly data plans via PHs. To successfully realize the PH-market, we face another two challenges besides usage randomness: users' incentive to trade and their random mobility to enter or leave the PH-range. To provide strong incentives for users to participate and trade in the PH-market, the trading price should be designed carefully to balance the data supply and demand and ensure the reduction of all users' costs. In addition, users are mobile and their arrival or departure will affect the relationship between supply and demand in the PH-market, making it difficult for users to estimate their costs. Our main contributions are summarized as follows:

- *A novel PH-market for data plan trading*: We propose a PH-market for heterogeneous users to trade their data plan monthly as sellers or buyers. We propose and analyze a market-clearing price to balance the data supply and demand in the PH-market. We show that the PH-market exists when user types are diverse in subscribed data plan or usage behavior.
- *Users' behaviors under random mobility*: Despite users' mobility to enter or leave the PH-range and the PH-market's variation, we analyze users' optimal trading data amounts and show that their average monthly costs reduce in the PH-market.
- *WSP's countermeasures to PH-market*: As the WSP's revenue decreases due to the user-initiated PH-market, we provide two user-acceptable countermeasures for the WSP to reduce its revenue loss – reducing the selling users' data quota and/or increasing the buying users' data quota. For the situations when there are more than one WSPs competing with each other, we discuss the evolution of the WSPs' data plans in the PH-market and show that at the equilibrium, the WSPs compete to set their data plans to meet all customers' maximum data usage against the PH-market.

To our best knowledge, this paper is the first work discussing user-initiated data plan trading by leveraging PH connections. There are some other works that consider generic user cooperation through incentive design, and two types of incentive mechanisms called reputation-based and market-based (or credit-based) systems have been investigated. In reputation-based systems, users gain trust as rewards by cooperation and receive punishments for uncooperative behaviors (e.g., [4], [5]). In market-based (or credit-based) systems, users receive payments (or credits) by cooperating with other users (e.g., [6]–[10]). Motivated by the 2CM (2nd exchange market) data trading platform introduced by China Mobile Hong Kong[1], there have been a handful of works dealing with leftover data trading to improve data users' payoffs via WSP-coordinated market [11], [12]. In the WSP-provided market investigated in [11], [12], the WSP serves as a trading platform manager to match the supply and demand of data users by charging transaction fee.

[1]https://www.hk.chinamobile.com/tc/2cm_intro.html

Unlike these works, we propose a user-initiated market in which each user can freely trade his optimal data amount by leveraging PHs without the WSP's involvement. Moreover, we further explore the WSP's countermeasures to the user-initiated trading in the PH-market as well as studying the competition between WSPs. It is worth noting that today's WiFi connections are mostly encrypted, so are many smartphone Apps (e.g., Apple email, Alipay) [13]–[15]. Therefore, the security and the privacy of a user can be assumed to be guaranteed in the PH-market as users can encrypt the transmitted data and add encryption to data downloaded via PHs, same as in conventional WiFi.

The rest of this paper is organized as follows. The system model before introducing the PH-market is given in Section II. In Section III, the PH-market is formally introduced including the analysis of the user behavior and the trading price. In Section IV, we study the WSP's user-acceptable countermeasures to the PH-market. The equilibrium data plans for the case with two WSPs competing with each other are analyzed in Section V. Section VI concludes this paper.

II. System Model without PH-market

In this section, we provide the WSP and user models before introducing the PH-market. The key parameters which will be used throughout the paper are summarized in Table I.

TABLE I: Key Parameters

Symbol	**Description**
B_l, B_h	subscribed data quota of light users/heavy users
P_l, P_h	subscription fee of light users/heavy users
p	unit price for extra data
D_l, D_h	maximum monthly data usage of light users/heavy users
d_l, d_h	minimum monthly data usage of light users/heavy users
r	PH-range
λ_l, λ_h	the density of light users/heavy users
K_l, K_h	the random number of light users/heavy users within the PH-range r
ξ_l, ξ_h	the average number of light users/heavy users within the PH-range r
π	trading price

We consider two types of users according to their monthly data usage behaviors: K_l light-usage users and K_h heavy-usage users. Our model and analysis can be extended to more than two user types without major change of the key insights (see Appendix B for details). Each type of users know their monthly data usage distribution through historical records and experience. Each light user's monthly data usage x_l is a random variable with probability density function $f_l(x_l)$ on the possible usage range $d_l \leq x_l \leq D_l$, where d_l and D_l are the minimum and maximum monthly data usage of light users, respectively. Note that any two light users' realized usages are generally different at the end of each month. Similarly, each heavy user's monthly data usage x_h is a random variable with probability density function $f_h(x_h)$ on the possible usage range $d_h \leq x_l \leq D_h$. For analysis tractability, we assume each user type's usage follows a uniform distribution as in [12], [16], i.e., $f_l(x_l) = \frac{1}{D_l - d_l}$ and $f_h(x_h) = \frac{1}{D_h - d_h}$. One can imagine that a heavy user generally demands more data than a light user, thus, $d_l < d_h$, $D_l < D_h$, and thus $\frac{d_l + D_l}{2} < \frac{d_h + D_h}{2}$. Our results can be extended to Gaussian distribution for data usage without major change of engineering insights (see Appendix A for details).

Given there are only two user types, it is optimal for the WSP to offer at most two two-part tariff data plans with one targeting at each user type [17], i.e., (B_l, P_l, p) for light users and (B_h, P_h, p) for heavy users. We reasonably set the unit extra data prices for these two plans identical[2]. For example, two popular data plans provided by China Mobile in Beijing are (1 GB, CNY 50, CNY 0.29 per MB) and (3 GB, CNY 100, CNY 0.29 per MB)[3] with identical unit price. How different data plans, such as unlimited data plan, impact the PH-market is discussed in Appendix C.

Generally, the data quota B_l chosen by light users should be larger than their minimum monthly data usage d_l. To save budget, the data quota B_l also should not be larger than the light users' maximum monthly data usage D_l. Thus, we assume that $d_l \leq B_l \leq D_l$. By choosing data plan (B_l, P_l, p), a light user's realized monthly cost for using an arbitrary $x_l \in [d_l, D_l]$ amount of data is

$$C_l(x_l|(B_l, P_l, p)) = P_l + p(x_l - B_l)^+, \tag{1}$$

where $(x)^+ = \max(x, 0)$.

Since the light user's monthly data usage x_l is a random variable, by taking expectation of $C_l(x_l|(B_l, P_l, p))$ in (1) over any possible x_l, the expected monthly cost is

$$\mathbb{E}(C_l|(B_l, P_l, p)) = P_l + \frac{p(D_l - B_l)^2}{2(D_l - d_l)}. \tag{2}$$

[2]If the surcharge prices are different, our analysis can be extended. The resulting difference is that in the data trading study in Section III, a user with lower surcharge price may want to over-sell data to those with higher surcharge price.

[3]http://www.10086.cn/4G/zixuantc/bj/

Similar to the light user's case, we assume that $d_h \leq B_h \leq D_h$. By subscribing to the long-term data plan (B_h, P_h, p), a heavy user's realized monthly cost for using an arbitrary $x_h \in [d_h, D_h]$ amount of data is

$$C_h(x_h|(B_h, P_h, p)) = P_h + p(x_h - B_h)^+, \tag{3}$$

and his expected monthly cost is

$$\mathbb{E}(C_h|(B_h, P_h, p)) = P_h + \frac{p(D_h - B_h)^2}{2(D_h - d_h)}. \tag{4}$$

By collecting lump-sum fees and the surcharges on over-usage from the two types of users, the expected monthly revenue of the WSP is

$$R = K_l\left(P_l + \frac{p(D_l - B_l)^2}{2(D_l - d_l)}\right) + K_h\left(P_h + \frac{p(D_h - B_h)^2}{2(D_h - d_h)}\right). \tag{5}$$

According to (5), the WSP's expected revenue decreases with both B_l and B_h and increases with P_l, P_h and p. Intuitively, the WSP wants to induce over-usage charge on both types of users, and prefers high lump-sum fees and costly unit prices to increase its revenue. However, in practice, the WSP may not be able to freely decide data quotas and prices, depending on the users acceptance and market competition.

In the following sections, we introduce the PH-market, where users can trade data freely among themselves.

III. User-initiated Data Plan Trading via PH-market

In this section, we propose the PH-market by allowing users in the PH-range to trade data with each other. By creating a low-power PH, a host user can only share cellular data connection with those within the PH-range (around 50 meters). The PH range is determined based on the buying users' QoS requirement (on SNR and data rate) and affects the economic benefit for sharing. As buying users' QoS requirement reduces and allows weaker received power, PH-range increases to cover more buying users.

Based on the buying users' QoS requirement reflected by the minimum received power $\bar{P}_r$, we now determine the PH-range r, by ensuring the actual received power P_r is no smaller than

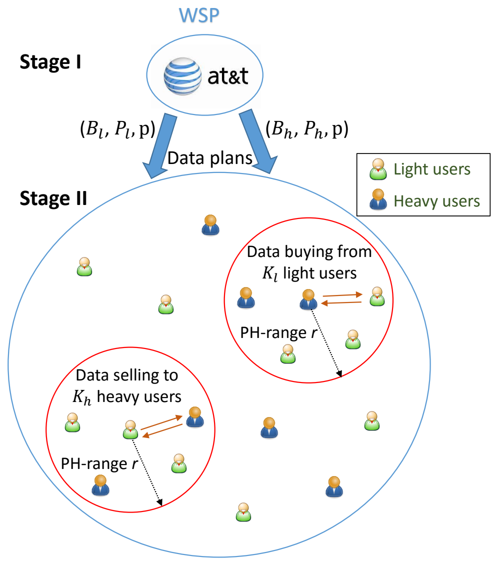


Fig. 1: Stackelberg game including one WSP and two types of users randomly distributed in the cellular network. A typical user can only trade data with users within the PH-range r. Both user types follow homogeneous Poisson point processes (PPPs) with user densities λ_l and λ_h, respectively

$\bar{P}_r$. We consider a typical signal propagation model by considering the pathloss[4], and received power at the buying user of distance d from the PH is

$$P_r = P_t c_0 \frac{d^{-\zeta}}{r_0^{-\zeta}}, \tag{6}$$

where P_t is the maximum PH's transmission power, c_0 is a constant equal to the pathloss at a reference distance r_0, and ζ is the pathloss exponent valued between 2 and 4.

We need to ensure that the received power P_r in (6) of the buying user at the PH cell-edge user with $d = r$ is still $\bar{P}_r$, we determine the maximum PH-range r as

$$r = \left(\frac{c_0 P_t}{\bar{P}_r}\right)^{\frac{1}{\zeta}} r_0. \tag{7}$$

From (7), a selling user will try to serve buying users within range r and a buying user will try to connect to a PH within r. The successful connection or trading event still depends on the users' mobility and population densities.

Suppose the light users and heavy users follow independent homogeneous Poisson point processes (PPPs) with user densities λ_l and λ_h, respectively (see Fig. 1). The number of neighboring

[4]Similar results of (6) and (7) can be obtained if more general fading channel is considered, where the required received power and maximum PH-range can be modified accordingly with a given maximum outage probability and fading distribution [18].

users inside the PH-range is random and can greatly affect the opportunity to trade data. Let K_l and K_h denote the random numbers of light users and heavy users within the PH-range r of a typical user (buyer or seller), respectively. As both user types' locations follow homogeneous PPP, K_l and K_h within the PH-range r follow Poisson distributions with means $\xi_l = \pi r^2 \lambda_l$ and $\xi_h = \pi r^2 \lambda_h$, respectively. Each mean formula is increasing in the PH-range r and the corresponding user density. The probability mass function of K_i is given by

$$\mathbb{P}_i(K_i) = \frac{\xi_i^{K_i}}{K_i!} e^{-\xi_i}, K_i \geq 0, i \in \{l, h\}. \tag{8}$$

We formulate the two-stage interaction between the WSP and heterogeneous users as a Stackelberg game (see Fig. 1), explained as follows:

- In Stage I, the WSP decides and charges data plans (lasting for long-term, e.g., two years[5]) (B_l, P_l, p) and (B_h, P_h, p) to the light users and heavy users in the cellular network before introducing the PH-market, respectively. Yet the WSP can still change them in a user-acceptable way after introducing the PH-market.
- In Stage II, the two types of users could choose to trade data with each other in the PH-market on the basis of the data plans provided by the WSP.

We use backward induction to first analyze users' behaviors in the PH-market at Stage II and then the WSP's countermeasure against the PH-market at Stage I.

As data usage is not realized yet, a user decides to buy or sell data under usage uncertainty and his belief on the other users' trading behavior. It is still possible to incur over-usage surcharge by the WSP. We denote the trading price per unit data as π. This trading price should be less than unit surcharge price p; otherwise, users will directly purchase data from the WSP once facing data deficit within each month.

In the following, given an arbitrary trading price π, we predict and analyze users' selling and buying behavior. To realize such trading, we then enforce a market-clearing price π^* by using the prediction of users' behavior to match supply with demand.

A. *Users' Trading Behavior in the PH-market Given Price π*

All light (heavy) users are symmetric and have the same decision in selling or buying data. As long as their costs are reduced, they will all participate in the PH-market. In this section, we

[5]http://info.singtel.com/personal/broadband-on-the-go/data-plans

analyze the situation when the light users are data sellers and heavy users are data buyers. For the situation when the light users are data buyers and heavy users are data sellers, the analysis is similar.

First, we analyze a light user's expected monthly cost when he acts as a seller in the PH-market. After selling data to heavy users, the remaining data quota may not be enough for the light user himself and he may need to buy data from the WSP eventually. Note that $d_l \leq B_l$; otherwise, the light user will not sell any data and the PH-market does not exist. Denote s as the amount of sold data to the PH-market. Due to the buying/heavy users' mobility, a selling/light user decides to sell data amount s continuously over time but cannot find any buyer with probability $\mathbb{P}_h(0) = e^{-\xi_h}$ within the PH-range r. The expected sold data of the light user is $(1 - e^{-\xi_h})s$[6]. To guarantee the basic data usage of the seller himself, the expected sold data should be less than the maximum leftover data, i.e., $(1 - e^{-\xi_h})s \leq B_l - d_l$. Then, by locally consuming any realized x_l amount of data, the monthly cost of the light user is given as

$$C_l(x_l|s, \pi) = P_l - \pi(1 - e^{-\xi_h})s + p(x_l - (B_l - (1 - e^{-\xi_h})s))^+. \tag{9}$$

Different from (1), $\pi(1 - e^{-\xi_h})s$ is the revenue received by the light user due to selling $(1 - e^{-\xi_h})s$ amount of data at price π and $p(x_l - (B_l - (1 - e^{-\xi_h})s))^+$ is the surcharge for over-usage (if any).

As $x_l \in [d_l, D_l]$ is random following uniform distribution, by taking expectation of $C_l(x_l|s, \pi)$ in (9) over any possible x_l, the expected monthly cost is

$$\mathbb{E}(C_l|s, \pi) = P_l - \pi(1 - e^{-\xi_h})s + \frac{p}{2(D_l - d_l)}(D_l - B_l + (1 - e^{-\xi_h})s)^2. \tag{10}$$

Since $\mathbb{E}(C_l|s, \pi)$ is convex with respect to s, by setting the derivative $d\mathbb{E}(C_l|s, \pi)/ds = 0$, we obtain the optimal shared data $s^*(\pi)$ shown in the following lemma.

Lemma 3.1: The optimal sold data $s^*(\pi)$ for the light user is given as

$$s^*(\pi) = \begin{cases} \frac{\frac{\pi(D_l - d_l)}{p} - D_l + B_l}{1 - e^{-\xi_h}}, \text{if } \pi > \frac{p(D_l - B_l)}{D_l - d_l}; \\ 0, \text{otherwise.} \end{cases} \tag{11}$$

And the optimal expected cost of the light user in the PH-market is

$$\mathbb{E}(C_l^*|\pi) = P_l + \pi(D_l - B_l) - \frac{\pi^2(D_l - d_l)}{2p}. \tag{12}$$

[6]Here we assume the there is enough data demand from the seller given buyer appearance. This will be ensured through our trading pricing as designed later.

The denominator of $s^*(\pi)$ tells that the selling/light users should oversell data to compensate for the revenue loss due to the probability of no buyers within the PH-range r. Note that $\frac{D_l-B_l}{D_l-d_l}$ is the probability for data-overage beyond B_l and $\frac{p(D_l-B_l)}{D_l-d_l}$ can be viewed as the light user's average penalty for over-usage. Thus, if the trading price π is large enough to cover the average penalty for over-usage $\frac{p(D_l-B_l)}{D_l-d_l}$, the light user would like to sell $(1-e^{-\xi_h})s^*(\pi)$ amount of data. Otherwise, the light user will not sell any data. From (11), the optimal sold data increases with the trading price π and quota B_l and decreases with the unit price p. Due to $\pi < p$, $(1-e^{-\xi_h})s^*(\pi) < B_l - d_l$ always holds in (11).

Then, we discuss a heavy user's expected monthly cost when he acts as a buyer in the PH-market. As heavy users cannot exactly predict their future data usage, the data bought from light users may not cover their monthly data usage. Thus, he may need to buy extra data from the WSP eventually. Note that $B_h \leq D_h$; otherwise, the heavy user will not buy any data and the PH-market does not exist. Denote b as the amount of purchased data from the PH-market. Due to the selling/light users' mobility, a buying/heavy user decides to buy data amount b continuously over time but cannot find any seller with probability $P_l(0) = e^{-\xi_l}$ within the PH-range r. The expected bought data of the heavy user is $(1-e^{-\xi_l})b$. Note that $(1-e^{-\xi_l})b$ should be less or equal to $D_h - B_h$, i.e., $(1-e^{-\xi_l})b \leq D_h - B_h$, since each buyer's data usage is upper-bounded by D_h. Then, in the PH-market, the monthly cost of heavy user for locally consuming any realized x_h amount of data is given as

$$C_h(x_h|b,\pi) = P_h + \pi(1-e^{-\xi_l})b + p(x_h - (B_h + (1-e^{-\xi_l})b))^+. \tag{13}$$

Here, $\pi(1-e^{-\xi_l})b$ is the payment to light users from buying $(1-e^{-\xi_l})b$ amount of data at price π and $p(x_h - (B_h + (1-e^{-\xi_l})b))^+$ is the surcharge for over-usage (if any).

As the heavy user's data usage $x_h \in [d_h, D_h]$ is random following uniform distribution, by taking expectation of $C_h(x_h|b,\pi)$ over any possible x_h, the expected monthly cost of heavy user is

$$\mathbb{E}(C_h|b,\pi) = P_h + \pi(1-e^{-\xi_l})b + \frac{p}{2(D_h-d_h)}(D_h - B_h - (1-e^{-\xi_l})b)^2. \tag{14}$$

Since $\mathbb{E}(C_h|b,\pi)$ is convex with respect to b, by setting the derivative $d\mathbb{E}(C_h|b,\pi)/db = 0$, we obtain the optimal bought data $b^*(\pi)$ shown in the following lemma.

Lemma 3.2: The optimal bought data $b^*(\pi)$ for the heavy user is given as

$$b^*(\pi) = \begin{cases} \frac{D_h - B_h - \frac{\pi(D_h-d_h)}{p}}{1-e^{-\xi_l}}, \text{if } \pi < \frac{p(D_h-B_h)}{D_h-d_h}; \\ 0, \text{otherwise.} \end{cases} \tag{15}$$

And the optimal expected cost of the heavy user in the PH-market is

$$\mathbb{E}(C_h^*|\pi) = P_h + \pi(D_h - B_h) - \frac{\pi^2(D_h - d_h)}{2p}. \tag{16}$$

The denominator of $b^*(\pi)$ tells that the buying/heavy users should overbuy data to compensate for the extra payment to the WSP due to the probability of no sellers within the PH-range r. Note that $\frac{D_h-B_h}{D_h-d_h}$ is the probability for data-overage beyond B_h and $\frac{p(D_h-B_h)}{D_h-d_h}$ can be viewed as the heavy user's average penalty for over-usage. Thus, the average penalty $\frac{p(D_h-B_h)}{D_h-d_h}$ should be larger than the unit trading price π to stimulate heavy users to buy $(1-e^{-\xi_l})b^*(\pi)$ amount of data in the PH-market. From (15), the optimal bought data decreases with respect to the trading price π and quota B_h and increases with the unit data price p. Obviously, $(1-e^{-\xi_l})b^*(\pi) \le D_h - B_h$.

Remark 3.1: From (12) and (16), we can conclude that if the light/heavy users act as buyers/sellers, the structure of their optimal expected costs in the PH-market is the same as (12) and (16), respectively.

B. Market-clearing Price and PH-market Existence

According to Lemmas 3.1 and 3.2, if $\frac{p(D_l-B_l)}{D_l-d_l} < \pi < \frac{p(D_h-B_h)}{D_h-d_h}$, the light users and heavy users would like to join the PH-market and act as sellers and buyers, respectively. Similarly, we can show that if $\frac{p(D_h-B_h)}{D_h-d_h} < \pi < \frac{p(D_l-B_l)}{D_l-d_l}$, the light users and heavy users act as buyers and sellers, respectively. Yet prior analysis of trading behavior is based on the assumption that the data supply always equals data demand. Thus, to ensure this assumption holds and the prior analysis is feasible, we decide a market-clearing price by a balance between the average total supply and the average total demand:

$$\sum_{K_l=1}^{\infty} \mathbb{P}_l(K_l)K_l s_e^*(\pi) = \sum_{K_h=1}^{\infty} \mathbb{P}_h(K_h)K_h b_e^*(\pi), \tag{17}$$

where $s_e^*(\pi) = (1 - e^{-\xi_h})s^*(\pi)$ and $b_e^*(\pi) = (1 - e^{-\xi_l})b^*(\pi)$ are the expected sold data of the light user and the expected bought data of the heavy user, respectively.

Note that K_l and K_h follow Poisson distributions with means $\xi_l = \pi r^2 \lambda_l$ and $\xi_h = \pi r^2 \lambda_h$, respectively, thus, (17) is equivalent to

$$\lambda_l s_e^*(\pi) = \lambda_h b_e^*(\pi), \tag{18}$$

which shows the total supply per unit area is equal to the total demand per unit area.

Theorem 3.1: The equilibrium trading price for market-clearing is given as

$$\pi^*(\frac{\lambda_l}{\lambda_h}) = \frac{p((D_h - B_h) + \frac{\lambda_l}{\lambda_h}(D_l - B_l))}{(D_h - d_h) + \frac{\lambda_l}{\lambda_h}(D_l - d_l)}. \tag{19}$$

Furthermore, for any positive $\lambda_l > 0$ and $\lambda_h > 0$, the equilibrium trading price $\pi^*(\frac{\lambda_l}{\lambda_h})$ encourages all users to participate by ensuring the following inequality:

- If the light users act as sellers and heavy users act as buyers,

$$\frac{p(D_l - B_l)}{D_l - d_l} < \pi^*(\frac{\lambda_l}{\lambda_h}) < \frac{p(D_h - B_h)}{D_h - d_h}; \tag{20}$$

- If the light users act as buyers and heavy users act as sellers,

$$\frac{p(D_h - B_h)}{D_h - d_h} < \pi^*(\frac{\lambda_l}{\lambda_h}) < \frac{p(D_l - B_l)}{D_l - d_l}. \tag{21}$$

Proof: To derive the average total supply and demand, we take the expectation of $K_l s_e^*(\pi)$ and $K_h b_e^*(\pi)$ over K_l and K_h, respectively. Since $\sum_{K_l=0}^{\infty} \mathbb{P}_l(K_l)K_l = \pi r^2 \lambda_l$ and $\sum_{K_h=0}^{\infty} \mathbb{P}_h(K_h)K_h = \pi r^2 \lambda_h$, by setting $\sum_{K_l=0}^{\infty} \mathbb{P}_l(K_l)K_l s_e^*(\pi) = \sum_{K_h=0}^{\infty} \mathbb{P}_h(K_h)K_h b_e^*(\pi)$, we have $\pi^*(\frac{\lambda_l}{\lambda_h})$ as shown in (19). It is easy to check that for the situation when the light users are buyers and the heavy users are sellers, the equilibrium trading price is the same as (19).

Take derivative of $\pi^*(y)$ with respect to y, where $y = \frac{\lambda_l}{\lambda_h}$, we can conclude that: If $\frac{p(D_l-B_l)}{D_l-d_l} < \frac{p(D_h-B_h)}{D_h-d_h}$, then $\frac{d\pi^*(y)}{dy} < 0$, which means that $\pi^*(\lambda_l/\lambda_h)$ decreases as λ_l/λ_h increases. If $\frac{p(D_l-B_l)}{D_l-d_l} > \frac{p(D_h-B_h)}{D_h-d_h}$, $\pi^*(\lambda_l/\lambda_h)$ is a increasing function of λ_l/λ_h. Note that $0 < \lambda_l/\lambda_h < +\infty$. As $y \to 0$, $\pi^*(y) \to \frac{p(D_h-B_h)}{D_h-d_h}$. As $y \to +\infty$, $\pi^*(y) \to \frac{p(D_l-B_l)}{D_l-d_l}$. Thus, the proof is completed. ■

Theorem 3.1 illustrates that the trading price $\pi^*(\frac{\lambda_l}{\lambda_h})$ balances data supply and demand in the PH-market: it increases as more users would like to buy data and decreases as more users would like to sell data. As shown in Fig. 2, the equilibrium trading price decreases with the ratio λ_l/λ_h when the light users act as sellers and heavy users act as buyers.

In the following, we will use π^* and $\pi^*(\frac{\lambda_l}{\lambda_h})$ interchangeably when no confusion is caused.

The PH-market's existence condition is given in the following theorem.

Theorem 3.2: The PH-market's existence condition is $\frac{D_l-B_l}{D_l-d_l} \neq \frac{D_h-B_h}{D_h-d_h}$. Furthermore,

- If $\frac{D_l-B_l}{D_l-d_l} < \frac{D_h-B_h}{D_h-d_h}$, the light users act as data sellers and heavy users act as data buyers;
- If $\frac{D_l-B_l}{D_l-d_l} > \frac{D_h-B_h}{D_h-d_h}$, the heavy users act as data sellers and light users act as data buyers.

Proof: According to (20), when $\frac{D_l-B_l}{D_l-d_l} < \frac{D_h-B_h}{D_h-d_h}$, the light users would like to sell data and heavy users would like to buy data in the PH-market. From (21), when $\frac{D_l-B_l}{D_l-d_l} > \frac{D_h-B_h}{D_h-d_h}$, the light users serve as buyers and heavy users serve as sellers. When $\frac{D_l-B_l}{D_l-d_l} = \frac{D_h-B_h}{D_h-d_h}$, both of the

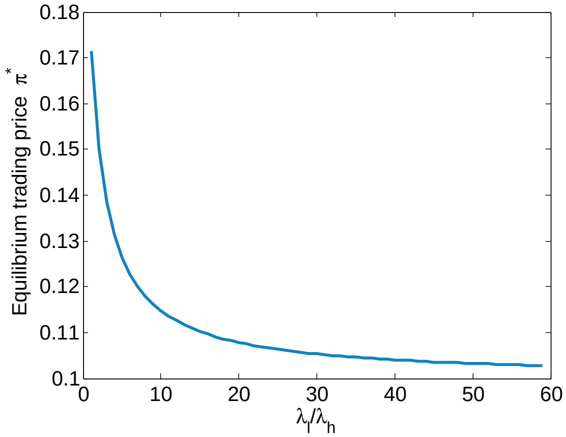


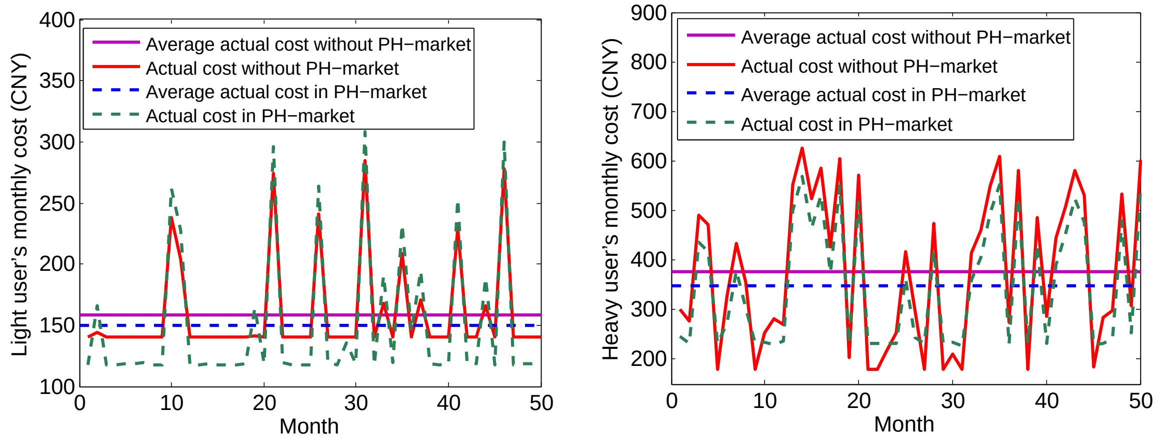
Fig. 2: Equilibrium trading price π^* versus λ_l/λ_h when the light users act as sellers and heavy users act as buyers

optimal sold data and bought data are 0, which implies that users will not trade data with each other and the PH-market does not exist. ■

The PH-market's existence condition when light users act as sellers and heavy users act as buyers is equivalent to $\frac{p(D_l-B_l)}{D_l-d_l} < \frac{p(D_h-B_h)}{D_h-d_h}$. Note that $\frac{p(D_l-B_l)}{D_l-d_l}$ can be viewed as the light user's average penalty for over-usage and $\frac{p(D_h-B_h)}{D_h-d_h}$ can be viewed as the heavy user's average penalty for over-usage. Therefore, the condition $\frac{p(D_l-B_l)}{D_l-d_l} < \frac{p(D_h-B_h)}{D_h-d_h}$ indicates that if the light user has a smaller penalty than the heavy user, it is worthwhile for the light users to sell data and heavy users to buy data in the PH-market at a middle price. Similarly, if $\frac{p(D_l-B_l)}{D_l-d_l} < \frac{p(D_h-B_h)}{D_h-d_h}$, i.e., the heavy user has a smaller penalty than the light user, then, the heavy users would like to be data sellers and light users would be data buyers.

C. Robustness of Trading Efficiency under User Mobility and Usage Randomness

In the above analysis, we have shown the reduction in users' expected monthly costs in the PH-market. In this section, we divide a month into many time slots and consider the user's actual monthly cost under the circumstance that users can trade data during a month randomly based on the number of neighboring users and the data demand/supply in each time slot. If the total data supply is larger than the total data demand in a certain time slot, the sellers can only sell the amount of required data to the buyers, and vice versa. When a user's monthly trading data reaches his expected trading amount (i.e., $(1-e^{-\xi_h})s^*(\pi)$ for the data sellers and $(1-e^{-\xi_l})b^*(\pi)$ for the data buyers) derived from Lemma 3.1 and 3.2, he will not trade data this month any



(a) light user

(b) heavy user

Fig. 3: The typical user's actual monthly cost

more. In the following simulation, we show that the typical user's average actual monthly cost in the PH-market is smaller than that without PH-market.

Consider an example where there are 60 users per unit area. We assume $\lambda_h : \lambda_l = 3 : 7$, i.e., $\lambda_h = 18$ users per unit area and $\lambda_l = 42$ users per unit area. Suppose heavy and light users subscribe to the data plan (B_l, P_l, p) =(3 GB, CNY 140, CNY 0.3 per MB) and (B_h, P_h, p) =(6 GB, CNY 180, CNY 0.3 per MB), respectively. Set $D_l = 3.5$ GB, $d_l = 2$ GB, $D_h = 7.5$ GB, $d_h = 5.5$ GB. Each user's actual monthly data usage is generated randomly according to a uniform distribution. Given the trading price π^*, it is shown in Fig. 3(a) and Fig. 3(b) that the typical light user's (seller) or heavy user's (buyer) actual cost in the PH-market may be larger than his actual cost without PH-market for a certain month. But his average actual monthly cost reduces in PH-market compared to that without PH-market.

IV. WSP's Countermeasures to PH-market

All users benefit from data plan trading and balancing in the proposed PH-market, but the WSP can no longer surcharge users for data over-usage as much as the case without data trading. In this section, we will investigate the WSP's countermeasures to the PH-market in Stage I. Without loss of generality, we consider the WSP's monthly revenue in one unit area in the following sections, i.e, on average λ_l light users and λ_h heavy users trade data in the PH-market.

By trading data at price π^* given in (19), the WSP's expected monthly revenue per unit area changes from (5) to

$$R_{PH} = \lambda_l \mathbb{E}(C_l^*|\pi^*) + \lambda_h \mathbb{E}(C_h^*|\pi^*). \tag{22}$$

Since $\mathbb{E}(C_l^*|\pi^*) < \mathbb{E}(C_l|(B_l, P_l, p))$ and $\mathbb{E}(C_h^*|\pi^*) < \mathbb{E}(C_h|(B_h, P_h, p))$, we have $R_{PH} < R$.

Equation (22) tells that the WSP can reduce its revenue loss due to PH-market by simply raising the subscription fee P_l, P_h and surcharge price p or decreasing the subscribed quota B_l and B_h to result in more data over-usages among users. However, such data plan change will violate the existing contracts with users and users can hardly agree. In order to let the users accept the contract change, the WSP should ensure the reduction for the users' expected monthly costs while increasing its expected revenue.

In the following, we discuss some user-acceptable strategies for the WSP to defend the PH-market and reduce its revenue loss due to the PH-market under the situation when light users are data sellers and heavy users are data buyers in the PH-market originally, i.e., $\frac{D_l - B_l}{D_l - d_l} < \frac{D_h - B_h}{D_h - d_h}$. For the situation when $\frac{D_l - B_l}{D_l - d_l} > \frac{D_h - B_h}{D_h - d_h}$ at the first place, the analysis is similar with some minor modifications.

Remark 4.1: For the case with one data plan, i.e., $P_l = P_h = P, B_l = B_h = B$, the WSP cannot find any user-acceptable strategy to reduce its revenue loss due to PH-market, no matter it decides to keep users in the PH-market or drive them out of the market.

A. Acceptable Reduction of Data Supply in PH-market

As the light users may have leftover data and account for the data supply in the PH-market, one possible countermeasure strategy for the WSP is to decrease each light user's data quota from B_l to B_l^-. To ensure light users' agreement for contract change, the WSP also reduce the lump-sum fee P_l to P_l^- such that each light user's expected cost is further reduced compared to that in the PH-market.

Since the initial status of the PH-market is $\frac{D_l - B_l}{D_l - d_l} < \frac{D_h - B_h}{D_h - d_h}$, from the PH-market's existence condition $\frac{D_l - B_l}{D_l - d_l} \neq \frac{D_h - B_h}{D_h - d_h}$, we can see that decreasing B_l can drive the users out of the PH-market. If B_l is further decreased such that $\frac{D_l - B_l}{D_l - d_l} > \frac{D_h - B_h}{D_h - d_h}$, all users join the PH-market again and the light users serve as buyers and heavy users serve as sellers. However, the WSP's expected revenue decreases after joining the PH-market. Therefore, when B_l is reduced to $B_l = D_l - \frac{D_h - B_h}{D_h - d_h}(D_l - d_l)$, i.e., $\frac{D_l - B_l}{D_l - d_l} = \frac{D_h - B_h}{D_h - d_h}$, the WSP will not further reduce B_l to let $\frac{D_l - B_l}{D_l - d_l} > \frac{D_h - B_h}{D_h - d_h}$.

The mechanism design of the data plan is shown as follows.

After decreasing B_l, if $B_l > D_l - \frac{D_h - B_h}{D_h - d_h}(D_l - d_l)$, i.e., $\frac{D_l - B_l}{D_l - d_l} < \frac{D_h - B_h}{D_h - d_h}$, then light users are still data sellers and heavy users are still data buyers in the PH-market. Since only the light user's data plan is revised, the WSP only needs to consider the light users' acceptance to this change. Therefore, the WSP aims to find B_l^*, P_l^* to maximize R_{PH} and decrease the expected monthly costs of the light users by a reasonably small constant $\varepsilon > 0$,[7] i.e., for an $\varepsilon > 0$,

$$\max_{B_l, P_l} R_{PH}(B_l, P_l) \tag{23}$$

$$\text{s.t.} \quad \mathbb{E}(C_l^* | \pi^*) - \mathbb{E}C_l^{PH} \leq -\varepsilon, \tag{24}$$

where R_{PH} is given in (22) and $\mathbb{E}C_l^{PH}$ is the original cost of light users in the PH-market.

From (24), the relationship between P_l and B_l is given as

$$P_l \leq -\varepsilon + \mathbb{E}C_l^{PH} + \frac{(\pi^*)^2(D_l - d_l)}{2p} - \pi^*(D_l - B_l). \tag{25}$$

According to (22), the WSP's expected revenue increases with P_l, thus, the expected revenue that the WSP aims to maximize becomes

$$R_{PH}^-(B_l) = \lambda_l\Bigg(\mathbb{E}C_l^{PH} + \frac{(\pi^*)^2(D_l - d_l)}{p} - \pi^*(D_l - B_l) - \varepsilon\Bigg) + \lambda_h\Bigg(P_h + \frac{(\pi^*)^2(D_h - d_h)}{2p}\Bigg). \tag{26}$$

Since R_{PH}^- is a concave function with respect to B_l, by setting $\frac{dR_{PH}^-}{dB_l} = 0$, we obtain B_l^* that maximizes R_{PH}^- shown in the following lemma.

Lemma 4.1: The optimal monthly data cap B_l^* for the light user provided by the WSP is given as

$$B_l^* = D_l - \frac{D_h - B_h}{D_h - d_h}(D_l - d_l). \tag{27}$$

The WSP's optimal data plan design for the light users is given as follows.

Proposition 4.1: To reduce the revenue loss due to PH-market and decrease the light user's expected cost simultaneously, the optimal data plan for the light user provided by the WSP is given as: for a reasonably small $\varepsilon > 0$,

$$B_l^- = D_l - \frac{D_h - B_h}{D_h - d_h}(D_l - d_l), \tag{28}$$

[7] If ε is large, which implies that the users' expected monthly costs are reduced a lot, the WSP's revenue may be smaller than that before adjusting the data plan. Therefore, the WSP would reduce the users' expected costs by a reasonably small constant ε to make users accept the data plan change while ensuring its own revenue increase. The upper bound of ε can be calculated easily by ensuring the WSP's expected revenue after adjusting the data plan is larger than that before adjusting the data plan.

$$P_l^- = -\varepsilon + \mathbb{E}C_l^{PH} - \frac{p(D_l - B_l^-)^2}{2(D_l - d_l)}, \tag{29}$$

which drives the users out of the PH-market.

Under the optimal data plan, the light user's expected cost is

$$\mathbb{E}(C_l|(B_l^-, P_l^-, p)) = \mathbb{E}C_l^{PH} - \varepsilon, \tag{30}$$

and the WSP's expected revenue is given as

$$R^- = \lambda_l(\mathbb{E}C_l^{PH} - \varepsilon) + \lambda_h\bigg(P_h + \frac{p(D_h - B_h)^2}{2(D_h - d_h)}\bigg). \tag{31}$$

Proof: Note that $R_{PH}^-(B_l)$ decreases with B_l when $B_l \geq D_l - \frac{D_h - B_h}{D_h - d_h}(D_l - d_l)$. Therefore, the WSP would like to set $B_l^- = B_l^* = D_l - \frac{D_h - B_h}{D_h - d_h}(D_l - d_l)$ to maximize its expected revenue, which drives the users out of the PH-market since $\frac{D_l - B_l^*}{D_l - d_l} = \frac{D_h - B_h}{D_h - d_h}$. Then, the subscription fee P_l^-, the light user's expected monthly cost $\mathbb{E}(C_l|(B_l^-, P_l^-, p))$ and the WSP's expected revenue R^- can be calculated easily. Actually, we can check that $R_{PH}^-(B_l)$ is identical to R^- when $B_l = D_l - \frac{D_h - B_h}{D_h - d_h}(D_l - d_l)$. ■

Remark 4.2: Even though the WSP's expected revenue is improved by adjusting the light user's data plan, it is still less than its original expected revenue without PH-market. This is because the light user's expected cost is reduced in the PH-market, i,e., $\mathbb{E}C_l^{PH} < \mathbb{E}(C_l|(B_l, P_l, p))$. Thus, $\mathbb{E}C_l^{PH} - \varepsilon < \mathbb{E}(C_l|(B_l, P_l, p)) = P_l + \frac{p(D_l - B_l)^2}{2(D_l - d_l)}$ for any $\varepsilon > 0$.

Besides presenting analytical results, we also use numerical results with practical data to illustrate the impact of the data plan change to WSP's revenue.

1) Numerical Results: Consider the same system as shown in Section III-C. We can calculate the expected monthly cost of the light user and heavy user before trading data are CNY 165 and CNY 348.75, respectively. Since $\pi^* = 0.1455$ CNY per MB, $\frac{p(D_l - B_l)}{D_l - d_l} = 0.1$, $\frac{p(D_h - B_h)}{D_h - d_h} = 0.225$, $\frac{p(D_l - B_l)}{D_l - d_l} < \pi^* < \frac{p(D_h - B_h)}{D_h - d_h}$ is satisfied. Therefore, light users act as sellers and heavy users act as buyers. The expected monthly cost of the light user and heavy user are CNY 159.83 and CNY 327.66, respectively, which are less than their expected monthly costs before joining PH-market.

In this example, we set $\varepsilon = 2.26$. It is observed in Fig. 4 that the WSP's expected revenue increases as B_l decreases in the PH-market and achieves its maximum value when $B_l = D_l - \frac{D_h - B_h}{D_h - d_h}(D_l - d_l)$. When the WSP reduces B_l to $B_l^- = D_l - \frac{D_h - B_h}{D_h - d_h}(D_l - d_l) = 2.375$ GB and P_l to $P_l^- = 31.01$ CNY, its expected monthly revenue becomes CNY 12895.65, which is larger than its original expected monthly revenue CNY 12610.91 after PH-market trading but is still less than its original expected revenue CNY 13207.5 before PH-market trading. Furthermore,

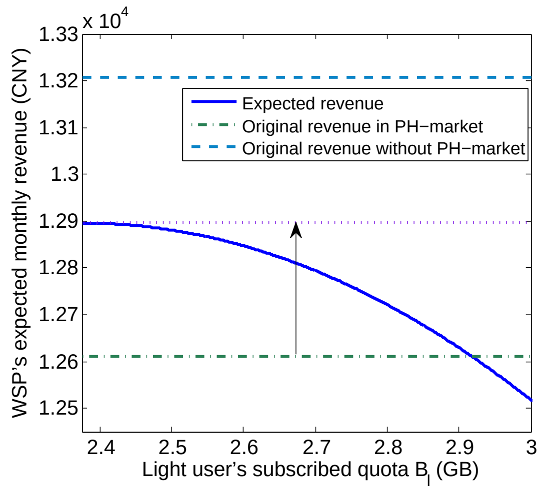


Fig. 4: WSP's expected monthly revenue versus light user's subscribed quota B_l

we can check that the expected monthly cost of the light user decreases to CNY 157.57, which is less than his original cost for both the situations with or without PH-market.

B. Acceptable Reduction of Data Demand in PH-market

Since heavy users may not have adequate data quota and account for the data demand in the PH-market, the WSP could increase each heavy user's subscribed quota from B_h to B_h^+ to prevent them from trading data with light users. In order to improve the expected revenue, the WSP also increases the subscription fee from P_h to P_h^+, and at the same time, guarantees that the heavy users' expected monthly costs are reduced compared to their original expected costs in the PH-market.

According to the initial status of the PH-market $\frac{D_l-B_l}{D_l-d_l} < \frac{D_h-B_h}{D_h-d_h}$, increasing B_h to $D_h - \frac{D_l-B_l}{D_l-d_l}(D_h-d_h)$ can drive the users out of the PH-market. Moreover, if B_h is further increased such that $\frac{D_l-B_l}{D_l-d_l} > \frac{D_h-B_h}{D_h-d_h}$, users trade data again and the heavy users become sellers and light users become buyers. As a result, the WSP's expected revenue decreases in the PH-market. Therefore, the WSP will not further increase B_h such that $B_h > D_h - \frac{D_l-B_l}{D_l-d_l}(D_h-d_h)$ to let users trade data again once the PH-market is driven out.

After increasing B_h, if $B_h < D_h - \frac{D_l-B_l}{D_l-d_l}(D_h-d_h)$, i.e., $\frac{D_l-B_l}{D_l-d_l} < \frac{D_h-B_h}{D_h-d_h}$, the roles of the light users and the heavy users remain unchanged. As only the heavy user's data plan is modified, the WSP only considers the heavy users' acceptance to this change. Therefore, the WSP aims to find B_h^* and P_h^* to maximize R_{PH} without increasing the heavy user's expected monthly cost, i.e., for an $\varepsilon > 0$,

$$\max_{B_h, P_h} R_{PH}(B_h, P_h) \tag{32}$$

$$\text{s.t.} \quad \mathbb{E}(C_h^*|\pi^*) - \mathbb{E}C_h^{PH} \leq -\varepsilon, \tag{33}$$

where R_{PH} is given in (22) and $\mathbb{E}C_h^{PH}$ is the original cost of heavy users in the PH-market.

According to (33), the relationship between P_h and B_h is given as

$$P_h \leq \mathbb{E}C_h^{PH} - \varepsilon + \frac{(\pi^*)^2(D_h - d_h)}{2p} - \pi^*(D_h - B_h). \tag{34}$$

Since the WSP's expected revenue R_{PH} increases with P_h, thus, the expected revenue that the WSP aims to maximize becomes

$$\begin{aligned} &R_{PH}^+(B_h) \\ =&\lambda_l\left(P_l + \frac{(\pi^*)^2(D_l - d_l)}{2p}\right) + \lambda_h\left(\mathbb{E}C_h^{PH} - \varepsilon + \frac{(\pi^*)^2(D_h - d_h)}{p} - \pi^*(D_h - B_h)\right). \end{aligned} \tag{35}$$

Since R_{PH}^+ is a concave function with respect to B_h, by setting $\frac{dR_{PH}^+}{dB_h} = 0$, we obtain B_h^* that maximizes R_{PH}^+ shown in the following lemma.

Lemma 4.2: The optimal monthly data cap B_h^* for the heavy user provided by the WSP is given as

$$B_h^* = D_h - \frac{D_l - B_l}{D_l - d_l}(D_h - d_h). \tag{36}$$

The WSP's optimal data plan design for the heavy users is given as follows.

Proposition 4.2: To reduce the revenue loss due to PH-market and decrease the heavy user's expected cost simultaneously, the optimal data plan for the heavy user provided by the WSP is given as: for a reasonably small $\varepsilon > 0$,

$$B_h^+ = B_h^* = D_h - \frac{D_l - B_l}{D_l - d_l}(D_h - d_h), \tag{37}$$

$$P_h^+ = -\varepsilon + \mathbb{E}C_h^{PH} - \frac{p(D_h - B_h^+)^2}{2(D_h - d_h)}, \tag{38}$$

which drives the users out of the PH-market.

Under the optimal data plan, the heavy user's expected cost is

$$\mathbb{E}(C_h|(B_h^+, P_h^+, p)) = \mathbb{E}C_h^{PH} - \varepsilon, \tag{39}$$

and the WSP's expected revenue is given as

$$R^+ = \lambda_l\left(P_l + \frac{p(D_l - B_l)^2}{2(D_l - d_l)}\right) + \lambda_h(\mathbb{E}C_h^{PH} - \varepsilon). \tag{40}$$

The proof is similar to that of Proposition 4.1 and is thus omitted.

V. Competitive WSPs' countermeasures to PH-market

In previous sections, we have studied the WSP's countermeasures to the PH-market under the condition that there is only one WSP existing in the PH-market. In reality, there may exist more than one WSPs providing a variety of data plans, e.g., two major WSPs in China: China Mobile and China UniCom. In this section, we consider the situation when there are two WSPs each offering one data plan to wireless users. Suppose WSP 1 offers data plan (B_l, P_l, p) to light users and WSP 2 offers data plan (B_h, P_h, p) to heavy users and the light users act as sellers and heavy users act as buyers in the PH-market at the first place, i.e., $\frac{D_l-B_l}{D_l-d_l} < \frac{D_h-B_h}{D_h-d_h}$.[8]

If WSPs 1 and 2 are fully cooperative to maximize the total revenue, the analysis and results are the same as in Section IV. We are thus interested in how the two WSPs compete with each other and what the impact of the PH-market is on them. In this section, we analyze competitive WSPs' countermeasure to PH-market.

Before introducing the PH-market, the WSP 1's expected monthly revenue is

$$R^1 = \lambda_l \left(P_l + \frac{p(D_l - B_l)^2}{2(D_l - d_l)} \right), \tag{41}$$

and the WSP 2's expected revenue is

$$R^2 = \lambda_h \left(P_h + \frac{p(D_h - B_h)^2}{2(D_h - d_h)} \right). \tag{42}$$

After the PH-market appears, the WSP 1's expected revenue becomes

$$R^1_{PH} = \lambda_l \left(P_l + \frac{(\pi^*)^2(D_l - d_l)}{2p} \right), \tag{43}$$

and WSP 2's expected revenue becomes

$$R^2_{PH} = \lambda_h \left(P_h + \frac{(\pi^*)^2(D_h - d_h)}{2p} \right). \tag{44}$$

Note that after introducing the PH-market, the WSP 1's expected revenue increases as the light users would pay more surcharge fee due to the data sold to heavy users, and the WSP 2's expected revenue decreases as the heavy users would buy less data from it. That is to say, the PH-market benefits the WSP whose customers serve as sellers in the PH-market but exerts a detrimental effect on the WSP whose customers serve as buyers.

In the following, we will analyze the WSPs' response to the PH-market.

[8]For the case when $\frac{D_l-B_l}{D_l-d_l} > \frac{D_h-B_h}{D_h-d_h}$ at the first place, the analysis is similar with some minor modifications.

A. WSP 2's Countermeasure to PH-market

As the WSP 1's expected revenue increases in the PH-market, it will not violate its data plan first. However, the WSP 2's expected revenue decreases in the PH-market. In order to reduce the revenue loss due to the PH-market and ensure heavy users' acceptance to the contract change, the WSP 2 aims to revise the data plan as $(\widehat{B_h^*}, \widehat{P_h^*}, p)$ to maximize its expected revenue without any adversely effect on heavy users at the beginning of the first month.

Lemma 5.1: The optimal monthly data cap $\widehat{B_h^*}$ for the heavy users offered by WSP 2 is given as

$$\widehat{B_h^*} = \min\{D_h - \frac{(D_l - B_l)(D_h - d_h)}{2(D_l - d_l)} + \frac{\lambda_l(D_l - B_l)}{2\lambda_h}, D_h\}, \tag{45}$$

which satisfies $\frac{D_l - B_l}{D_l - d_l} \geq \frac{D_h - \widehat{B_h^*}}{D_h - d_h}$.

Then $\widehat{P_h^*}$ can be set accordingly as: for a reasonably small $\varepsilon > 0$,

$$\widehat{P_h^*} = \overline{\mathbb{E}C_h^{PH}} - \varepsilon + \frac{(\pi^*)^2(D_h - d_h)}{2p} - \pi^*(D_h - \widehat{B_h^*}), \tag{46}$$

where $\overline{\mathbb{E}C_h^{PH}}$ is the expected cost of heavy users in the PH-market at previous month.

Proof: See Appendix D. ■

After the WSP 2 adjusts its data plan to $(\widehat{B_h^*}, \widehat{P_h^*}, p)$, the heavy users and light users exchange roles and WSP 2 gains benefit in the PH-market. However, the WSP 1's expected revenue decreases under such change and it could increase B_l to resist it.

B. WSP 1's Response to WSP 2's Countermeasure

In order to reduce the revenue loss due to WSP 2's data plan change, the WSP 1 aims to revise the data plan as $(\widehat{B_l^*}, \widehat{P_l^*}, p)$ to maximize its expected revenue without any adversely effect on light users at the beginning of the second month.

Lemma 5.2: The optimal monthly data cap $\widehat{B_l^*}$ for the light users offered by WSP 1 is given as

$$\widehat{B_l^*} = \min\{D_l - \frac{(D_h - B_h)(D_l - d_l)}{2(D_h - d_h)} + \frac{\lambda_h(D_h - B_h)}{2\lambda_l}, D_l\}, \tag{47}$$

which satisfies $\frac{D_l - \widehat{B_l^*}}{D_l - d_l} \leq \frac{D_h - B_h}{D_h - d_h}$.

Then $\widehat{P_l^*}$ is calculated accordingly as: for a reasonably small $\varepsilon > 0$,

$$\widehat{P_l^*} = -\varepsilon + \overline{\mathbb{E}C_l^{PH}} + \frac{(\pi^*)^2(D_l - d_l)}{2p} - \pi^*(D_l - \widehat{B_l^*}), \tag{48}$$

where $\overline{\mathbb{E}C_l^{PH}}$ is the expected cost of light users in the PH-market at previous month.

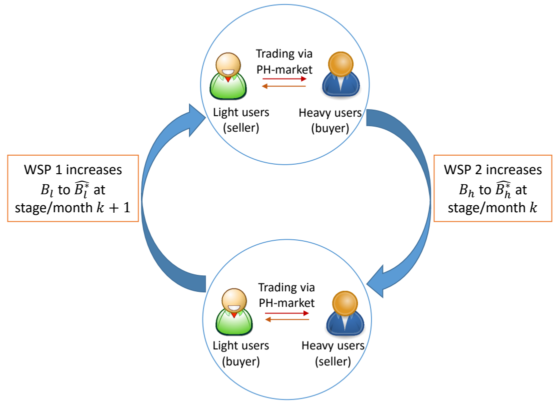


Fig. 5: WSP's data plan updating process

The proof is similar to that of Lemma 5.1 and thus we omit it here.

After the WSP 1 modifies its data plan to $(\widehat{B_l^*}, \widehat{P_l^*}, p)$, the light users and heavy users exchange roles again, and the WSP 1 gains benefit in the PH-market while the WSP 2's expected revenue decreases.

C. Convergence of Competitive WSPs' Data Plans

According to the above discussion, when light users act as sellers and heavy users act as buyers, the WSP 2 increases B_h to $\widehat{B_h^*}$ given in (45) to maximize its expected revenue and exchange the roles of the light users and heavy users at the beginning of month k. Then, WSP 1 increases B_l to $\widehat{B_l^*}$ shown in (47) to exchange the roles of the heavy users and light users at month $k+1$. The WSP's data plan updating process is shown in Fig. 5. The user-acceptable equilibrium data plans for the two WSPs case is summarized in the following theorem.

Theorem 5.1: For the two-WSPs case, the data plan updating process is repeated until $B_h = D_h$ and $B_l = D_l$, and the user-acceptable equilibrium data plans for the WSP 1 and WSP 2 are $(D_l, \widehat{P_l^*}, p)$ and $(D_h, \widehat{P_h^*}, p)$, respectively, where $\widehat{P_l^*}$ and $\widehat{P_h^*}$ can be calculated through (48) and (46), respectively. In addition, from any initial data plans, the equilibrium data plans can be achieved at most 3 months.

Proof: According to (45) and (47), when $B_l = D_l$ and $B_h = D_h$, we have $\widehat{B_h^*} = D_h$ and $\widehat{B_l^*} = D_l$, which indicates that the equilibrium points are achieved. Then, $\widehat{P_l^*}$ and $\widehat{P_h^*}$ are calculated through (48) and (46), respectively. Since $\frac{D_l - \widehat{B_l^*}}{D_l - d_l} = \frac{D_h - \widehat{B_h^*}}{D_h - d_h} = 0$, the PH-market does not exist at the equilibrium data plan.

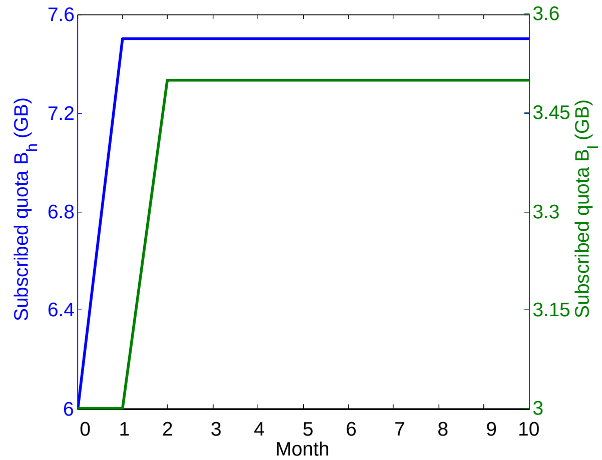
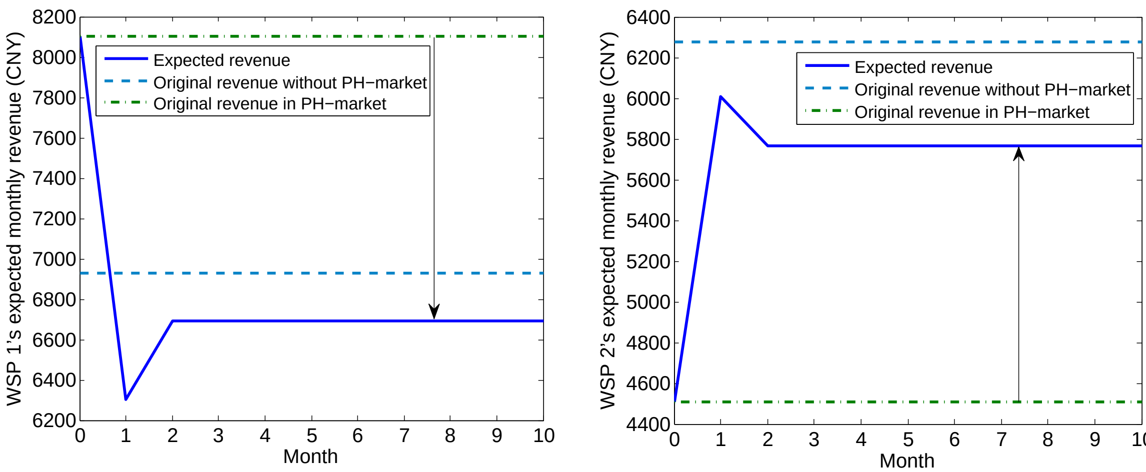


Fig. 6: Evolution of WSPs' data plans



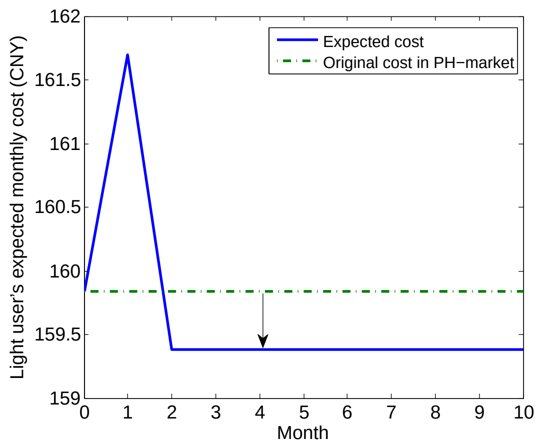
(a) WSP 1

(b) WSP 2

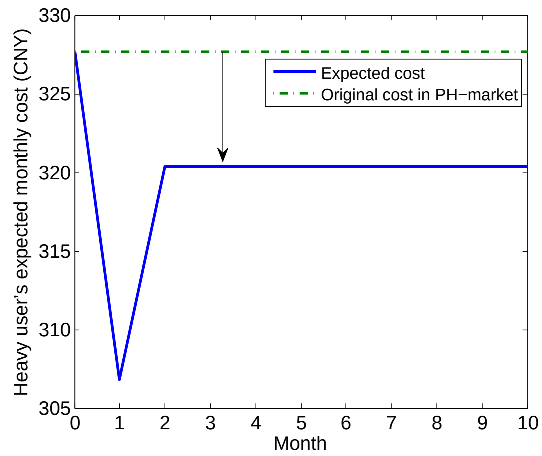
Fig. 7: Evolution of WSP's expected revenue

Note that if $-\frac{D_h-d_h}{D_l-d_l}+\frac{\lambda_l}{\lambda_h}>0$, then $-\frac{D_l-d_l}{D_h-d_h}+\frac{\lambda_h}{\lambda_l}<0$. Therefore, according to (45) and (47), $\widehat{B_h^*}=D_h$ at first month or third month and $\widehat{B_l^*}=D_l$ at the second month, which means that the equilibrium data plans can be achieved at 2 months or 3 months. ■

At the equilibrium data plans, both WSP 1 and WSP 2 set the subscribed quota B_l, B_h as the maximum monthly data usage of light users and heavy users, i.e., D_l, D_h, respectively. Therefore, the subscribed quota is adequate for all users and the PH-market no longer exists.



(a) light user



(b) heavy user

Fig. 8: Evolution of user's expected monthly cost

D. Numerical Results

Consider the same system as shown in Section III-C. Starting from the initial data plans, it is shown in Fig. 6 that the equilibrium data plans are achieved within 2 months. At the equilibrium data plans, WSP 1's expected monthly revenue decreases while WSP 2's expected monthly revenue increases compared to their original expected monthly revenues in the PH-market (see Fig. 7(a) and Fig. 7(b). However, the expected revenues of both WSP 1 and WSP 2 cannot reach their original revenues without PH-market. After updating the data plans, the expected monthly costs of the light users and heavy users are reduced (see Fig. 8(a) and Fig. 8(b)).

VI. Conclusion

This paper studies the data trading between cellular users enabled by WiFi personal hotspots. In the PH-market we proposed, we analyze all users' expected costs by taking users' random data usage, incentive to trade, and user mobility into account and a market-clearing price is derived. Furthermore, we also explore how the WSP adjusts the data plans to reduce its revenue loss due to the PH-market by user-acceptable strategies for the cases with two data plans and competing WSPs, respectively. One of our possible future work is to extend to the static trading price to the dynamic pricing, yet this can be bothering to users, as he not only needs to check his usage over time but also the dynamic trading price. Another future direction could be users' data plan changing.

APPENDICES

A. Gaussian Distribution for Data Usage

In this subsection, we show that our results also apply to other distribution for data usage. Consider the case when the light user's and heavy user's monthly data usage x_l, x_h follow a Gaussian distribution with mean μ_l, μ_h and variance σ_l^2, σ_h^2, respectively. By checking the first-order condition of the user's expected monthly cost, we have the following results.

Lemma 6.1: The optimal sold data $\bar{s}^*(\bar{\pi})$ for the light user is

$$\bar{s}^*(\bar{\pi}) = \begin{cases} \frac{B_l - \mu_l - \sqrt{2}\sigma_l \mathrm{erf}^{-1}(1 - \frac{2\bar{\pi}}{p})}{1 - e^{-\xi_h}}, \text{if } \bar{\pi} > (1 - \mathrm{erf}(\frac{B_l - \mu_l}{\sqrt{2}\sigma_l}))\frac{p}{2}; \\ 0, \text{otherwise.} \end{cases} \tag{49}$$

where $\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$.

And the optimal bought data $\bar{b}^*(\bar{\pi})$ for the heavy user is

$$\bar{b}^*(\bar{\pi}) = \begin{cases} \frac{\sqrt{2}\sigma_h \mathrm{erf}^{-1}(1 - \frac{2\bar{\pi}}{p}) - B_h + \mu_h}{1 - e^{-\xi_l}}, \text{if } \bar{\pi} < (1 - \mathrm{erf}(\frac{B_h - \mu_h}{\sqrt{2}\sigma_h}))\frac{p}{2}; \\ 0, \text{otherwise.} \end{cases} \tag{50}$$

Theorem 6.1: The equilibrium trading price for market-clearing under Gaussian distribution is given as

$$\bar{\pi}^*(\frac{\lambda_l}{\lambda_h}) = \left(1 - \mathrm{erf}\Big(\frac{\frac{\lambda_l}{\lambda_h}(B_l - \mu_l) + B_h - \mu_h}{\sqrt{2}\sigma_h + \sqrt{2}\sigma_l \frac{\lambda_l}{\lambda_h}}\Big)\right)\frac{p}{2}. \tag{51}$$

For any positive $\lambda_l, \lambda_h > 0$, the equilibrium trading price $\bar{\pi}^*(\frac{\lambda_l}{\lambda_h})$ encourages all users to participate by ensuring the following inequality:

- If the light users act as sellers and heavy users act as buyers,

$$\left(1 - \mathrm{erf}(\frac{B_l - \mu_l}{\sqrt{2}\sigma_l})\right)\frac{p}{2} < \bar{\pi}^*(\frac{\lambda_l}{\lambda_h}) < \left(1 - \mathrm{erf}(\frac{B_h - \mu_h}{\sqrt{2}\sigma_h})\right)\frac{p}{2}; \tag{52}$$

- If the light users act as buyers and heavy users act as sellers,

$$\left(1 - \mathrm{erf}(\frac{B_h - \mu_h}{\sqrt{2}\sigma_h})\right)\frac{p}{2} < \bar{\pi}^*(\frac{\lambda_l}{\lambda_h}) < \left(1 - \mathrm{erf}(\frac{B_l - \mu_l}{\sqrt{2}\sigma_l})\right)\frac{p}{2}. \tag{53}$$

The proof is similar as the case under uniform distribution and thus omitted here.

When the light users act as sellers and heavy users act as buyers, the equilibrium trading price decreases with the ratio λ_l/λ_h, which coincides with the situation under uniform distribution (see Fig. 2). It is shown in Fig. 9 that when the light users act as sellers and heavy users act as buyers, the light user's equilibrium expected cost decreases as the trading price increases (more heavy users would like to buy data), while the heavy user's equilibrium expected cost increases with

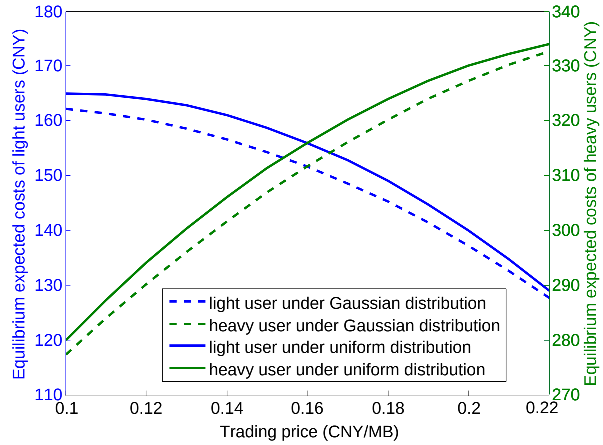


Fig. 9: Light users' and heavy users' equilibrium expected costs versus trading price

the trading price. Moreover, as shown in Fig. 9, the light/heavy user's equilibrium expected cost under Gaussian distribution is less than that under uniform distribution when the mean and variance of the light/heavy user's monthly data usage under both distributions are the same. This is because the user's monthly data usage concentrates mainly on its mean under Gaussian distribution, thus the users can better estimate their trading data amount compared with uniform distribution. Consequently, their expected monthly costs reduce due to less over-purchase/over-supply.

According to Theorem 6.1, we have the following theorem.

Theorem 6.2: The PH-market's existence condition under Gaussian distribution is $\frac{B_l-\mu_l}{\sigma_l} \neq \frac{B_h-\mu_h}{\sigma_h}$. Furthermore,

- If $\frac{B_l-\mu_l}{\sigma_l} > \frac{B_h-\mu_h}{\sigma_h}$, the light users act as data sellers and heavy users act as data buyers;
- If $\frac{B_l-\mu_l}{\sigma_l} < \frac{B_h-\mu_h}{\sigma_h}$, the heavy users act as data sellers and light users act as data buyers.

Note that $\frac{B_l-\mu_l}{\sigma_l}$ and $\frac{B_h-\mu_h}{\sigma_h}$ can be viewed as the light user's and heavy user's average leftover data, respectively. Therefore, if the light user has larger leftover data than the heavy user, i.e., $\frac{B_l-\mu_l}{\sigma_l} > \frac{B_h-\mu_h}{\sigma_h}$, the light users would like to be data sellers and the heavy user would like to be data buyers, and vice versa.

B. Heterogeneous Mix of Users

In this subsection, we show that our model can accommodate a more heterogeneous mix of users who are no longer symmetric in either light or heavy user type. Consider the case when there are a set of heterogeneous users $\mathcal{U}$ and each user chooses data plan $(B_i, P_i, p), i \in \mathcal{U}$.[9] Suppose each user's monthly data usage $x_i, i \in \mathcal{U}$ is a random variable following a uniform distribution on the possible usage range $d_i \leq x_i \leq D_i$. Without much loss of generality, we do not model $\mathcal{U}$ with any mobility processes and assume they are connected within PH-range r.

First, we analyze the case when there are three heterogeneous users – light user, heavy user, median user with $\frac{D_l-B_l}{D_l-d_l} \leq \frac{D_m-B_m}{D_m-d_m} < \frac{D_h-B_h}{D_h-d_h}$. For the cases with more heterogeneous users and other possible relationships between all users, the analysis is similar with some minor modifications.

Proposition 6.1: The equilibrium market-clearing price for three heterogeneous users with $\frac{D_l-B_l}{D_l-d_l} \leq \frac{D_m-B_m}{D_m-d_m} < \frac{D_h-B_h}{D_h-d_h}$ is given as

$$\tilde{\pi}^* = \frac{p(D_h - B_h + D_l - B_l + D_m - B_m)}{D_l - d_l + D_m - d_m + D_h - d_h}, \tag{54}$$

which encourages the light user and heavy user to participate in the PH-market and act as data seller and data buyer, respectively.

- If $\tilde{\pi}^* > \frac{p(D_m-B_m)}{D_m-d_m}$, the median user acts as data seller;
- If $\tilde{\pi}^* < \frac{p(D_m-B_m)}{D_m-d_m}$, the median user acts as data buyer;
- If $\tilde{\pi}^* = \frac{p(D_m-B_m)}{D_m-d_m}$, the median user will not join the PH-market.

Proof: All users' optimal trading data can be derived directly according to Lemmas 3.1 and 3.2. Then, by letting the total supply be equal to the total demand, we have the equilibrium market-clearing price $\tilde{\pi}^*$. We can verify that $\frac{p(D_l-B_l)}{D_l-d_l} < \tilde{\pi}^* < \frac{p(D_h-B_h)}{D_h-d_h}$, thus, the light user acts as data seller and heavy user acts as data buyer. By checking the relationship between $\tilde{\pi}^*$ and $\frac{p(D_m-B_m)}{D_m-d_m}$, the role of the median user can be determined. ■

Then, the PH-market's existence condition for the general case is summarized as follows.

Theorem 6.3: The PH-market's existence condition for a set of heterogeneous users is that, there exist at least two users such that $\frac{D_i-B_i}{D_i-d_i} \neq \frac{D_j-B_j}{D_j-d_j}$, where $i, j \in \mathcal{U}$ and $i \neq j$.

To prove Theorem 6.3, similarly as Proposition 6.1, we can analyze the cases with more than three heterogeneous users and other possible relationships between all users. Then, we can conclude that if there are at least two users such that $\frac{D_i-B_i}{D_i-d_i} \neq \frac{D_j-B_j}{D_j-d_j}, i \neq j \in \mathcal{U}$, there always exist users who are willing to join the PH-market to trade data.

[9]Similar to Section II, we look at the same unit price for extra data as an operator usually decides the same unit extra data price for different data plans. Note that if the surcharge prices are different, the buying users always buy data from the user with cheapest surcharge price if over-use.

C. Impact of Unlimited Data Plan

In this subsection, we discuss how the unlimited data plan impacts the PH-market. We consider the situation when the heavy users subscribe to the unlimited data plan (B_h, P_h) with $B_h = +\infty$ and no over-usage surcharge and the light users subscribe to the two-part tariff data plan (B_l, P_l, p). Then we expect heavy users to sell unused data while light users buy from them[10]. Note that if $B_l > D_l$, the light users have adequate data and will not buy any data from the heavy users. To avoid this trivial case, in the following, we consider $d_l \leq B_l \leq D_l$.

According to Lemma 3.2, the optimal bought data $b^*(\pi)$ for the light user is given as

$$b^*(\pi) = \begin{cases} \frac{D_l - B_l - \frac{\pi(D_l - d_l)}{p}}{1 - e^{-\xi_h}}, \text{if } \pi < \frac{p(D_l - B_l)}{D_l - d_l}; \\ 0, \text{otherwise.} \end{cases} \tag{55}$$

For each of the heavy users, he aims to find an optimal trading price π_u^* to maximize his profit while ensuring positive demand from light users, that is,

$$\max_{\pi} \pi b^*(\pi) \tag{56}$$

$$\text{s.t.} \quad \pi < \frac{p(D_l - B_l)}{D_l - d_l}. \tag{57}$$

Since $\pi b^*(\pi)$ is a concave function with respect to π. Using the first order condition, we obtain π_u^* that maximizes the heavy users' profits shown in the following lemma.

Lemma 6.2: The optimal trading price determined by the heavy users is given as

$$\pi_u^* = \frac{p(D_l - B_l)}{2(D_l - d_l)}. \tag{58}$$

Before introducing the PH-market, the WSP's expected monthly revenue per unit area is

$$\bar{R} = \lambda_h P_h + \lambda_l \left(P_l + \frac{p(D_l - B_l)^2}{2(D_l - d_l)} \right). \tag{59}$$

After the PH-market appears, the WSP's expected monthly revenue becomes

$$\begin{aligned} \bar{R}_{PH} =& \lambda_h P_h + \lambda_l \left(P_l + \frac{(\pi_u^*)^2 (D_l - d_l)}{2p} \right) \\ =& \lambda_h P_h + \lambda_l \left(P_l + \frac{p(D_l - B_l)^2}{8(D_l - d_l)} \right), \end{aligned} \tag{60}$$

which is less than $\bar{R}$.

[10]For the situation when the light users subscribe to the unlimited data plan and the heavy users subscribe to the two-part tariff data plan, the analysis is similar.

Similar as Section IV-B, the WSP could increase the light users' subscribed quota and subscription fee simultaneously to prevent them from trading data with heavy users and improve its expected monthly revenue. Meanwhile, to make the light users accept the contract change, the WSP should ensure the reduction for the light user's expected monthly cost compared to that in the PH-market.

Proposition 6.2: To reduce the revenue loss due to PH-market and decrease the light user's expected cost simultaneously, the WSP sets the optimal data plan for the light user as follows to drive the users out of the PH-market: for a reasonably small $\varepsilon > 0$,

$$\bar{B}_l^* = D_l, \tag{61}$$

$$\bar{P}_l^* = \mathbb{E}C_l^{PH} - \varepsilon, \tag{62}$$

where $\mathbb{E}C_l^{PH}$ is the original cost of light users in the PH-market.

Under the optimal data plan, the light user's expected cost is

$$\mathbb{E}(C_l|(\bar{B}_l^*, \bar{P}_l^*, p)) = \mathbb{E}C_l^{PH} - \varepsilon, \tag{63}$$

and the WSP's expected revenue is given as

$$\bar{R}^+ = \lambda_l(\mathbb{E}C_l^{PH} - \varepsilon) + \lambda_h P_h. \tag{64}$$

The proof is similar to that of Lemma 4.1 and Proposition 4.1 and is thus omitted.

Under the optimal data plan for light users, the WSP sets the subscribed quota B_l as the light user's maximum monthly data usage D_l. Therefore, the subscribed quota is adequate for all users and the PH-market no longer exists.

D. *Proof of Lemma 5.1*

If the heavy users still act as buyers in the PH-market after WSP 2 changes the data plan, i.e., $B_h < D_h - \frac{D_l - B_l}{D_l - d_l}(D_h - d_h)$, the expected revenue that WSP 2 aims to maximize is R_{PH}^2. Then, for an $\varepsilon > 0$, we have

$$\max_{B_h, P_h} R_{PH}^2 \tag{65}$$

$$\text{s.t.} \quad \mathbb{E}(C_h^*|\pi^*) - \overline{\mathbb{E}C_h^{PH}} \leq -\varepsilon, \tag{66}$$

where $\overline{\mathbb{E}C_h^{PH}}$ is the expected cost of heavy users in the PH-market at previous month. Note that at the first month, $\overline{\mathbb{E}C_h^{PH}} = \mathbb{E}C_h^{PH}$. This problem is equivalent to finding B_h^* to maximize

$$\widetilde{R_{PH,<}^2} = \lambda_h\left(\overline{\mathbb{E}C_h^{PH}} - \varepsilon + \frac{(\pi^*)^2(D_h - d_h)}{p} - \pi^*(D_h - B_h)\right). \tag{67}$$

However, it is easy to check that $\frac{(\pi^*)^2(D_h-d_h)}{p}-\pi^*(D_h-B_h)$ is always negative when $\frac{D_l-B_l}{D_l-d_l} < \frac{D_h-B_h}{D_h-d_h}$. Thus, $\widetilde{R^2_{PH,<}} < \lambda_h(\overline{\mathbb{E}C_h^{PH}}-\varepsilon)$. To drive out the PH-market, the WSP 2 sets $B_h = D_h - \frac{D_l-B_l}{D_l-d_l}(D_h-d_h)$ and P_h is set as $P_h = -\varepsilon + \overline{\mathbb{E}C_h^{PH}} - \frac{p(D_h-B_h)^2}{2(D_h-d_h)}$ for a positive ε to ensure the cost reduction of the heavy users. Then, the expected cost of the heavy user is $\mathbb{E}(C_h|(B_h,P_h,p)) = \overline{\mathbb{E}C_h^{PH}} - \varepsilon$ and the WSP 2's expected revenue is $\widetilde{R^2} = \lambda_h(\overline{\mathbb{E}C_h^{PH}}-\varepsilon)$. If B_h is further increased such that $D_h - \frac{D_l-B_l}{D_l-d_l}(D_h-d_h) < B_h \le D_h$, users trade data again and the heavy users act as sellers and light users act as buyers.

Similar to the analysis when heavy users are sellers, in this case, WSP 2 aims to find a $\widehat{B_h^*}$ to maximize

$$\widetilde{R^2_{PH,>}} = \lambda_h\bigg(\overline{\mathbb{E}C_h^{PH}} - \varepsilon + \frac{(\pi^*)^2(D_h-d_h)}{p} - \pi^*(D_h-B_h)\bigg). \tag{68}$$

When $\frac{D_l-B_l}{D_l-d_l} > \frac{D_h-B_h}{D_h-d_h}$, we can check that $\widetilde{R^2_{PH,>}} > \lambda_h(\overline{\mathbb{E}C_h^{PH}}-\varepsilon)$. Since $\widetilde{R^2_{PH,<}} < \widetilde{R^2} < \widetilde{R^2_{PH,>}}$, WSP 2 would like to increase B_h to $\widehat{B_h^*}$ to let $\frac{D_l-B_l}{D_l-d_l} > \frac{D_h-B_h}{D_h-d_h}$ at the beginning of the first month. Since $\frac{d^2\widetilde{R^2_{PH,>}}}{dB_h^2} < 0$, taking the derivative of (68) with respect to B_h, the optimal monthly data cap $\widehat{B_h^*}$ offered by WSP 2 is given as (45). The proof of Lemma 5.1 is thus completed. ■

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