A study on carrier-removal techniques in fringe projection profilometry

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Abstract

This paper describes a comparison of several carrier-removal techniques normally used in fringe projection for surface shape measurement. The performance of various algorithms, including the necessity for manual intervention, the complexity in data recording, and the side effect on measurement accuracy, is evaluated. The applicability of nonlinear carrier removal and the restrictions on the direction of carrier fringe in various algorithms are also investigated in this paper. It is also shown that an advanced algorithm is able to achieve carrier removal with minimal manual intervention and significantly simplify the calibration process of a fringe projection system.

Keywords: Carrier fringe; Carrier removal; Fringe projection profilometry

1. Introduction

In fringe projection profilometry [1,2], a carrier fringe pattern is projected onto an object surface to encode depth information. A deformed fringe pattern can generally be expressed as

\[ I(x, y) = I_0(x, y) + I_M(x, y) \cos[2\pi f(x, y) + \phi(x, y)], \]  

where \( I \) is the recorded intensity; \( x \) and \( y \) are spatial variables; \( I_0 \) and \( I_M \) are the background and modulation intensities, respectively; \( f \) is the frequency of the carrier fringe; and \( \phi \) is a depth-related phase angle. Phase extraction algorithms, such as Fourier transform [1,3] and phase shifting [2,4,5], can be used to extract wrapped phase data from one or several fringe patterns. A phase unwrapping process could subsequently retrieve a continuous phase distribution containing both shape-related phases \( \phi(x, y) \) and carrier fringe-related phase components \( 2\pi f(x, y) \).

In an optical measurement process, it is essential to establish the relationship between a phase value and the height of an object point. The phase-to-height conversion is also referred to as the calibration of a system. Several researchers [6–9] have proposed direct calibration methods, in which the phase-to-height relationship is obtained from system geometrical parameters without the necessity to remove the carrier phase component. Zhou and Su [6] proposed implementing the phase-to-height conversion based on two geometrical parameters obtained by measurement of at least three different parallel reference planes. An unwrapped phase value composed of the carrier and shape-related components was directly converted to an actual height. However, as relatively few geometrical parameters are considered, the method is only applicable when the carrier fringe is in a specific direction.

Salsa et al. [7] reported a coordinate transform scheme for system calibration. The method incorporates a minimization process to quantify up to seven unknown parameters. Due to the large number of parameters considered, the analysis is generally applicable to diverse system geometries. However, estimation of the unknowns is computationally involved and the minimization algorithm relies on manual intervention to prevent it from being trapped in an erroneous solution. A more practical strategy was subsequently proposed by Pavageau et al. [8]. Instead of using a minimization process to quantify all the system geometrical parameters, Pavageau et al. suggested estimating only three of the most important parameters from an unwrapped phase map and directly measuring the rest. The disadvantage of this approach is that measurement error would reduce the overall accuracy. Another valuable approach was proposed by Servin M. et al. [9], the
method integrated wrapped phase extraction, phase unwrapping, and carrier removal into a single process. However, due to the highly iterative and automatic data-processing strategy, the algorithm might produce misleading results when the signal-to-noise ratio is relatively low. As can be seen from the above methods, direct system calibration without prior removal of the carrier is either experimentally inconvenient or computationally unstable. An alternative approach to the phase-to-height conversion that has been continually studied is the retrieval of linearity between phase and height, which is also called carrier phase component removal. In this paper, several carrier-removal techniques, namely spectrum-shift [3], average-slope [10], plane-fitting, reference-subtraction [1], phase-mapping [11], and series-expansion [12,13] methods, are reviewed and compared. Depending on the linearity of the carrier phase function \(2\pi f(x,y)\), the carrier can be classified into two types: linear carrier with \(f(x,y)\) being the first-order function of \(x\) or \(y\), and nonlinear carrier with \(f(x,y)\) being a higher-order function of \(x\) or \(y\). Furthermore, if \(f(x,y)\) is related to only \(x\) or \(y\), the carrier fringe is in a specific direction and is called a one-dimensional (1-D) carrier, while if \(f(x,y)\) is related to both \(x\) and \(y\), the carrier fringe is in an arbitrary direction and is called two-dimensional (2-D) carrier. It is shown that a primitive form of some existing techniques is confined to the 1-D carrier-removal problem; however, with appropriate modification, the techniques can be extended to a general situation of 2-D carrier removal.

2. Linear carrier removal

2.1. Spectrum-shift approach

Takeda et al. described the Fourier transform method for the fringe pattern analysis [3] and suggested that the carrier phase component could be removed in the frequency domain via a spectrum shift. Fig. 1(a) shows the frequency spectrum of a 1-D fringe signal with a carrier phase component given by

\[
2\pi f(x,y) = 2\pi f_xx,
\]  

where \(f_x\) is the frequency of the carrier fringe in the \(x\) direction. The center of the fundamental frequency component is shifted to the center of the frequency spectrum in Fig. 1(b). An inverse Fourier transform of the shifted spectrum would produce a phase distribution without the carrier phase component. The principle of this approach is based on the property of Fourier transform: a spectrum shift of distance \(-f_x\) in the frequency domain is equivalent to the subtraction of a linear component \(2\pi f_xx\) in the spatial domain. Based on Eq. (1), the spectrum shift exactly removes the linear carrier from an unwrapped phase map and retrieves information-related phases.

The spectrum-shift approach can be conveniently extended to 2-D carrier removal. If the carrier fringe is in an arbitrary direction, the carrier frequency can be expressed as

\[
f(x,y) = f_xx + f_yy,
\]  

where \(f_x\) and \(f_y\) are carrier frequency measured in the \(x\) and \(y\) directions, respectively. In this case, one needs to shift the fundamental frequency component by \(-f_x\) in the \(x\) direction and \(-f_y\) in the \(y\) direction. As an example, Fig. 2(a) shows a fringe pattern with carrier fringe in an arbitrary direction and Fig. 2(b) indicates the spectrum shift. After inverse Fourier transform, an unwrapped phase map can be obtained (Fig. 2(c)) with the carrier phase component removed.

Although this approach is theoretically correct, it is not accurate in detecting the frequency of a carrier fringe.
The discrete Fourier transform (DFT) could only measure \( f_x \) or \( f_y \) in terms of an integer number of pixels, while the actual value could be in a fractional pixel. Hence, the spectrum shift with integer pixel accuracy leaves a considerable amount of residual carrier phases unaffected. Another limitation of the method is that manual intervention is normally required to identify the fundamental frequency component and filtering window size.

2.2. Average-slope approach

In contrast with the spectrum-shift approach, Li et al. [10] proposed a carrier-removal technique based on spatial analysis approach. Basically, the method removes the carrier phases by subtracting the first derivative (average-slope) from an unwrapped phase map. According to Eq. (1), the first derivative of an unwrapped phase distribution contains the slope of a carrier and that of shape-related phases. Li et al. showed that the average of the first derivative over a phase map is a good approximation to the slope of the carrier phase component

\[
\frac{1}{N} \sum \frac{d(2\pi f_x + \phi)}{dx} = 2\pi f_x + \frac{1}{N} \sum \frac{d\phi}{dx},
\]

where \( N \) is the total number of pixels in the image. Since the positive slopes of the shape-related phases are cancelled out by the negative slopes, the average slope is zero. Hence, the subtraction of the average slope removes the carrier phase component. An advantage gained by using the average-slope approach is the fully automatic carrier-removal process. Manual intervention is completely avoided.

However, the accuracy of the carrier slope estimation is case dependent. If the slope of a specific shape-related phase distribution cannot be averaged out, there will be a tilt in the resultant phase map, which indicates an error in carrier removal. Also, the primitive form of the method is applicable to 1-D carrier only and improvement should be made to adapt the method to 2-D situations.

2.3. Plane-fitting approach

The plane-fitting approach is an extension of the average-slope method. It is applicable to 2-D carrier removal and its accuracy is independent of the features of the shape-related phase distribution. As shown in Eq. (3), a major task in the estimation of a linear carrier is to obtain the carrier frequency in the \( x \) and \( y \) directions.
These unknowns can be calculated based on a least squares plane-fitting strategy. A plane function can be expressed as

$$\phi_p(x, y) = 2\pi f_x x + 2\pi f_y y + \phi_{c,0},$$

where $\phi_c$ is the carrier phase function, $f_x$ and $f_y$ are as defined in Eq. (3), and $\phi_{c,0}$ is the initial phase value. An error function is defined as the square of the differences between the carrier phase function and the experimentally obtained phases:

$$\text{Er}(f_x, f_y, \phi_{c,0}) = \sum_{(x,y) \in U} [2\pi f_x x + 2\pi f_y y + \phi_{c,0} - \phi_{\text{c,exp}}(x, y)]^2,$$

(6)

where $\phi_{\text{c,exp}}$ refers to an experimentally obtained phase value on a reference plane. Taking the first partial derivative of Er with respect to $f_x$, $f_y$, and $\phi_{c,0}$, one can get three parallel equations, from which the unknowns $f_x$, $f_y$, and $\phi_{c,0}$ can be calculated. Subsequently, the carrier function can be determined and the 2-D carrier can be removed by subtracting the carrier function from the overall phase distribution.

The plane-fitting approach requires measured phase values on a reference plane for the plane-fitting process, since the phase value of a reference point contains only the carrier phase component. Hence the carrier-removal accuracy is not affected by the shape-related phase distribution. Moreover, the reference plane need not be measured separately from the object. A single measurement can be achieved by setting a reference plane to the background of an object, as shown in Fig. 2(a). In the unwrapped phase map, data points from the reference region are selected for the plane-fitting process.

In previous discussions, all three methods tackle the problem of carrier removal by an estimation of the carrier phase function. The spectrum-shift approach retrieves the slope of a carrier in the frequency domain, while the average-slope approach implements a spatial average. The plane-fitting method achieves the best accuracy among the three, as an incorporated least-squares process ensures the minimum estimation error.

In practical applications, the carrier fringe patterns will most likely contain non-uniform fringe spacing over an object surface. This renders the carrier phase function nonlinear to the spatial variables. In such a situation, the above methods are not applicable and advanced techniques for nonlinear carrier removal should be used.

3. Nonlinear carrier removal

3.1. Reference-subtraction approach

The reference-subtraction technique [1], proposed by Takeda and Mutoh, requires individual measurement of an object and a reference plane. The unwrapped phase maps of the object and reference plane are calculated. Figs. 3(a) and (b) show the 3-D plot of a fish model surface and a reference plane. As can be seen, the reference phase map contains only the carrier phase component, while the object phase map has both carrier and shape-related phases. Subsequently, the subtraction of the reference phase map from the object phase map gives the phase distribution of the object profile, as shown in Fig. 3(c).

The idea of reference subtraction is straightforward and the method is robust, whatever features of a carrier (1-D or 2-D, linear or nonlinear) are obtained by measurement of the reference plane. Hence, the method is generally applicable to diverse systems geometries. However, the disadvantage of using two sets of recordings hinders application on measurements that require a high data-recording speed. Another undesirable outcome of reference subtraction is that the phase measurement uncertainty is doubled in the subtraction process, as shown in the following equations:

$$\phi_{\text{obj}} = \phi_c + \phi_s \pm \phi_E,$$

(7)

$$\phi_{\text{ref}} = \phi_c \pm \phi_E,$$

(8)

where $\phi_{\text{obj}}$ and $\phi_{\text{ref}}$ represent the unwrapped phase maps of an object and reference plane, respectively; $\phi_c$ and $\phi_s$ represent the theoretical carrier-related and shape-related phase components, respectively; and $\phi_E$ is the maximum measurement phase error. The measurement uncertainty is described in the range $[-\phi_E, \phi_E]$. The subtraction of the reference phase map ($\phi_{\text{ref}}$) from the object phase map ($\phi_{\text{obj}}$) results in

$$\phi_{\text{obj}} - \phi_{\text{ref}} = \phi_s \pm 2\phi_E.$$

(9)

The maximum error is doubled because each point in the two measurements has different, random phase errors. Instead of being eliminated by subtraction, the error is magnified. Consequently, the overall uncertainty is increased.

3.2. Phase-mapping approach

To reduce the complexity in data recording, a phase-mapping approach was proposed by Srinivasan et al. [11]. The method does not require an additional measurement of a reference plane and therefore makes the alignment of an experimental system easier. A test object can be mounted on a reference plate and one measurement records all necessary information. When an unwrapped phase map is obtained, an arbitrary point on the object is mapped to a point with an identical phase value on the reference plane. The distance between two points on an image could be converted to the object height with known system geometrical parameters, such as projection angle, relative position of the projection and imaging optics.

The original presentation of the method is a phase-to-height conversion technique that omits the carrier-removal process. However, if the system geometrical parameters are unknown, the phase-mapping strategy can be modified into a carrier-removal method. Originally, two points with an identical phase value are mapped to obtain the difference in
their distance. The modified algorithm maps two points with identical position (in x or y) on the images to obtain a phase difference, which represents the object shape-related phase component. Fig. 4(a) shows an unwrapped phase map of a cylindrical model. All sectional phase data are mapped onto a section on the reference plane, as indicated by the arrow pointers. Since the carrier phase component is in the x direction, data points with identical x position are mapped. The phase difference between an object point and a corresponding reference point is the shape-related phase value. Fig. 4(b) shows a 3-D plot of the phase difference map. As can be seen, the object profile is obtained and the carrier phase component is removed. It is to be noted that the modified phase-mapping technique still has the problem of magnifying phase measurement uncertainty, since the mapping is carried on experimentally obtained data.

3.3. Series-expansion approach

A recently reported series-expansion approach aims at retrieving the actual carrier phase function by a series...
expansion [12,13]. The rigorous mathematical expression of a nonlinear carrier is normally complicated and related to various system geometrical parameters. Therefore, it is impossible to quantify unknown geometrical parameters directly. However, it is found that a carrier function can be approximated by a series expansion

$$\phi_c(x) = \sum_{n=0}^{\infty} a_n x^n,$$

(10)

$$\phi_c(x, y) = a_{00} + a_{10} x + \cdots + a_{N-1,0} x^{N-1} + a_{0,N} x^N + a_{1,1} x y + \cdots + a_{N-1,1} x^{N-1} y + a_{N,N} x^N y,$$

(11)

where $a_n$ and $a_{np}$ are coefficients of elements in the series. Eq. (10) is a series expansion of a 1-D carrier and Eq. (11) is that of a 2-D carrier. This indicates that whatever the form of a specific carrier function, it can be estimated by fitting a series function to experimentally obtain the phase data on a reference plane. Hence, the quantification of geometrical parameters in various situations can be unified into a simple process of curved line fitting (1-D carrier) or curved surface fitting (2-D carrier). Fig. 5(a) shows an unwrapped phase map of a lion model with nonlinear carrier phase values. If one takes care of the linear carrier component only, the resultant phase map shown in Fig. 5(b) will contain a residual carrier. The nonlinear carrier function should be estimated by a series expansion and Fig. 5(c) shows the corrected results.

A major advantage of the series-expansion approach is that it is generally applicable to different measurement geometries. Note that the plane-fitting approach for linear carrier removal is a special case of the series-expansion method. The series analysis also enables a high level of automation that requires little manual intervention. Furthermore, as a series function instead of measurement data is subtracted from the overall phase distribution, the measurement uncertainty will not be affected.

Table 1 shows a comparison of the capabilities of the various carrier-removal techniques. It can be seen that except for the average-slope and phase-mapping approaches, all the other techniques are capable of removing 2-D carriers. Among the techniques that can handle nonlinear carriers, only the series-expansion method does not magnify the phase measurement uncertainty. Generally, if the Fourier-transform method [1,3] is used to retrieve a phase map and a linear carrier is inherent in the system geometry, the spectrum-shift approach should be applied for carrier removal because it requires minimal additional computation. If the phase-shifting algorithm [2,4,5] is used to extract phase data, the plane-fitting approach becomes a better choice, since a least squares fitting process is normally faster and more accurate than the process of Fourier transform, filtering, and inverse Fourier transform.

As far as a nonlinear carrier is concerned, the series-expansion method should always be considered first, as it
outperforms other techniques in several aspects, such as experimental simplicity, accuracy, and automation. However, the series-expansion method is valid only when phase data on a reference plane are retrievable or the measured object has a self-reference. If these conditions cannot be satisfied, the reference-subtraction approach should be used. With an additional measurement of a reference plane, the retrieval of the carrier phase component is ensured.

4. Conclusions

This paper discusses several carrier-phase-component-removal techniques for fringe projection profilometry. Three methods, namely spectrum shift, average slope, and plane fitting for linear carrier removal, represent the basis of carrier estimation. The reference-subtraction and phase-mapping techniques are applicable to nonlinear carrier removal. Both methods retrieve a nonlinear carrier by using direct measurement data without solving the carrier function. However, measurement errors are introduced in the carrier-removal process. In addition, the phase-mapping method is not suitable for a 2-D carrier, since it is difficult to automatically determine a reference section perpendicular to the carrier fringe, which consists of complete information of a 2-D carrier. The series-expansion method avoids the technical difficulty of resolving a rigorous mathematical expression for a nonlinear carrier; instead, it approximates the carrier function by a series expansion. The deterministic feature of a series function guarantees that no additional measurement uncertainty will be introduced. It is shown in this paper that a good carrier-removal algorithm can retrieve the linearity between a shape-related phase and the actual object height with considerable accuracy. Subsequently, the calibration process simply becomes one of obtaining a linear translation coefficient for the phase-to-height conversion. This would result in great saving of time and effort compared to methods previously reported on the measurement or estimation of the geometrical parameters.

References