Shortcut to adiabatic light transfer in waveguide couplers with a sign flip in the phase mismatch

To cite this article: Wei Huang et al 2020 J. Phys. D: Appl. Phys. 53 035104

View the article online for updates and enhancements.
1. Introduction

Waveguide directional couplers are important elements in integrated optics, with applications in power transfer between waveguides [1], mode conversion [2], polarization rotation [3], quantum communication [4] and many other practical fields [5]. The electric field propagation in coupled waveguides can be accurately described within the coupled-mode theory (CMT) [6] and recently, it was shown that the spatial dynamics of coupled waveguides is analogous to the temporal dynamics of quantum optical systems driven by external electromagnetic fields [7, 8]. Building on this analogy between quantum mechanics and wave optics, many optical systems to manipulate the propagation [9] and polarization [10] of light were proposed based on quantum optical techniques.

The most widely used quantum optical method in designing directional waveguide couplers is adiabatic evolution. Several adiabatic techniques such as stimulated Raman adiabatic passage (STIRAP) [11] and rapid adiabatic passage (RAP) [12, 13] were used to achieve robust optical power switching between two, three and even an array of coupled waveguides [14–16]. Most recently, some elegant experimental papers showed an implementation of the optical waveguide coupler by using adiabatic following [17, 18]. The main advantage of these waveguide couplers is their robustness to parameter variations while maintaining the efficiency of power transfer at 100%. However, adiabatic evolution requires slow change in the coupling parameters and overall dynamics, which necessitates an impractically long device length due to the adiabaticity condition.
To speed up the adiabatic evolution, shortcut to adiabaticity (STA) was initially proposed in the context of quantum optical systems to produce the same final populations in a finite, shorter time [19–21]. More recently, STA techniques based on Lewis–Riesenfeld invariants were applied in wave optics to design short and robust coupled waveguides [22–24, 26]. Another STA approach based on transitionless quantum driving (counterdiabatic driving) [20, 27] was also used to realize directional waveguides couplers [24, 28] and beam splitters [29], where an additional coupling at maximum coupling between the waveguides was required.

In this paper, we apply counter-diabatic driving with unitary transformation to the phase mismatch model with a sign flip at maximum coupling. The coupling of the phase mismatch model has a hyperbolic-secant spatial shape while the phase mismatch is constant, with a sign flip at the coupling maximum. This model was previously used to design a two-waveguide coupler, which realizes complete achromatic all-optical switching [30]. Here, we apply counter-diabatic STA to this model to design a shorter and more robust directional coupler for two and three waveguides that notably does not require an increase in coupling strength. Therefore, compared to previous designs, we present a more compact and robust (against fabrication inaccuracies) optical switching device [30]. Besides, the quantum control model which we employ (STA with phase mismatch model) does not require reducing the minimum distance between input/output and the middle waveguide, thus minimizing the fabrication challenges [26].

Our paper is organized as follows. In section 2, we review the coupled mode theory (CMT) and the phase mismatch model as applied to adiabatic evolution in coupled waveguides. In section 3, we apply STA to design a two-waveguide coupler and in section 4 we present numerical results about its performance. In the following section 5, we extend the system to three coupled waveguides and use STA to propose a robust achromatic equal superposition beam splitter. Finally, we present our conclusions in section 6.

2. Adiabatic light transfer in coupled waveguides

We consider two evanescently coupled optical waveguides as shown in figure 1. Adopting the paraxial approximation, we describe the propagation of a monochromatic light beam in the waveguide structure of figure 1 in the framework of the CMT [7]. The spatial propagation of the electric field amplitudes $c_1(z)$ and $c_2(z)$ in the $z$ direction is governed by a set of coupled differential equations,

\[
\frac{d}{dz} \begin{bmatrix} c_1(z) \\ c_2(z) \end{bmatrix} = \begin{bmatrix} \Delta(z) & \Omega(z) \\ \Omega(z) & -\Delta(z) \end{bmatrix} \begin{bmatrix} c_1(z) \\ c_2(z) \end{bmatrix}, \tag{1}
\]

where $\Omega(z)$ is the coupling coefficient between the two waveguides and the phase mismatch $\Delta(z) = |\beta_1(z) - \beta_2(z)|/2$ is the difference between the corresponding propagation constants, $\beta_1(z)$ and $\beta_2(z)$. The absolute squares of the electric field amplitudes are the dimensionless light intensities in the waveguides, $I_{1,2}(z) = |c_{1,2}(z)|^2$, which are normalized to $I_1(z) + I_2(z) = 1$ in the lossless waveguides scenario.

We consider the phase mismatch coupling model, with coupling strength $\Omega(z) = \Omega_0\text{sech}(2\pi z/L)$. Furthermore, the phase mismatch is $\Delta(z) = -\Delta_0$ when $z < 0$ and $\Delta(z) = \Delta_0$ when $z > 0$, where $\Omega_0$ is the maximum coupling amplitude and $\Delta_0$ is a fixed phase mismatch. They are both positive and real. The total length of the waveguide structure is $2L$, while the middle point is at $z = 0$. This coupling model is known to be analytically solvable and was previously used in [32] to realize a complete population inversion in atomic systems. More recently, it was applied to achieve a complete light transfer between two waveguides and a beam splitter for three coupled waveguides [30].

With the help of the unitary transformation $U_0$ (in the diabatic basis),

\[
U_0 = \begin{bmatrix} \cos(\theta/2) & -\sin(\theta/2) \\ \sin(\theta/2) & \cos(\theta/2) \end{bmatrix}
\]

we transform the diabatic basis $\mathbf{c}(z) = [c_1(z), c_2(z)]^T$ to the adiabatic basis $\mathbf{a}(z) = [a_1(z), a_2(z)]^T$ according to $\mathbf{a}(z) = U_0^{-1}\mathbf{c}(z)$. The transformation $U_0$ is unitary and the mixing angle $\theta$ is defined as $\tan(\theta) = \Omega(z)/\Delta(z)$. The operator $H(z)$ in the adiabatic basis is given by $H_i(z) = U_0^{-1}H(z)U_0 - iU_0^{-1}\dot{U}_0$, where the overdot represents a derivative with respect to $z$. 

**Figure 1.** A schematic of the waveguide structure with length $2L$ that is used for complete achromatic optical switching. Two evanescently coupled waveguides made of slabs with refractive indexes $n_1$ and $n_2$ are embedded in a medium with an index of refraction $n_3$. A Gaussian-shaped light beam is initially injected in the left waveguide and at the end the waveguide structure it is switched to the right waveguide.
For the system evolution to follow the adiabatic path, the adiabatic condition must be fulfilled. That is, the difference between the diagonal elements of $H_d(z)$ must be much larger than the off-diagonal elements. Parametrically, the adiabatic condition is satisfied when

$$\frac{\dot{\phi}}{2} = \frac{\Omega \Delta - \hat{\Delta} \Omega}{2(\Omega^2 + \hat{\Delta}^2)} \ll \sqrt{\Omega^2 + \hat{\Delta}^2},$$

(3)

which means that $\Delta(z)$ and $\Omega(z)$ must vary slowly with the spatial parameter $z$. We note that in relation to coupled waveguides devices the adiabatic condition entails long device lengths for the realization of high fidelity adiabatic light transfer.

## 3. Shortcut to adiabatic light transfer in waveguides

We use the STA protocol to design optical switching devices with shorter characteristic lengths. The STA is achieved by introducing an additional coupling between the waveguides, described by $H_d(z)$, which is used to nullify the off-diagonal elements of the adiabatic operator $H_a(z)$. The additional coupling operator is $H_d(z) = i \sum_j \langle a_j | \partial_z a_j \rangle$, which in the basis $\mathbf{e}(z)$ is given by,

$$H_d(z) = \begin{bmatrix}
0 & -i \dot{\phi} / 2 \\
i \dot{\phi} / 2 & 0
\end{bmatrix}.$$

(4)

The total effective coupling operator is then $H_{\text{eff}}(z) = H(z) + H_d(z)$ with $H(z)$ being the coupling operator in the diabatic basis from equation (1). We thus obtain

$$H_{\text{eff}}(z) = \begin{bmatrix}
\Delta(z) & \Omega(z) - i \Omega_0(z) \\
\Omega(z) + i \Omega_0(z) & -\Delta(z)
\end{bmatrix},$$

(5)

where the additional coupling term is $\Omega_0(z) \equiv \dot{\phi} / 2$ and $\Omega(z)$ and $\Delta(z)$ are the coupling and phase mismatch of the phase mismatch model.

As the coupling from equation (5) is complex, which is not physical for a coupled waveguide system, we use the transformation matrix $U_1$,

$$U_{\phi} = \begin{bmatrix}
e^{-i \phi / 2} & 0 \\
0 & e^{i \phi / 2}
\end{bmatrix},$$

(6)

with $\tan(\phi) = \Omega_0(z) / \Omega(z)$, to transform $H_{\text{eff}}(z)$ such as to remove the phase from the coupling terms. We thus obtain,

$$H_{\text{eff}}(z) = \begin{bmatrix}
\Delta_{\text{eff}}(z) & \Omega_{\text{eff}}(z) \\
\Omega_{\text{eff}}(z) & -\Delta_{\text{eff}}(z)
\end{bmatrix},$$

(7)

where $\Omega_{\text{eff}}(z) = \sqrt{\Omega(z)^2 + \Omega_0(z)^2}$, and $\Delta_{\text{eff}}(z) = \Delta(z) - \phi(z)/2$.

If the additional coupling $\Omega_0(z)$ between the waveguides is strong enough, the effective Hamiltonian $H_{\text{eff}}(z)$ can in fact follow the adiabatic path in an arbitrary short time. However, there is a physical limitation stating that the additional modifying coupling cannot be larger than the original one, that is,

$$|\Omega_0(z)| \leq |\Omega(z)| \leq |\Omega_0(z)| \quad [19].$$

To check if the STA model fulfills this inequality, we turn our attention to the behavior of the coupling parameter at the phase mismatch point, $\dot{z} = 0$. It is easy to see that $\lim_{\dot{z} \to \pm \infty} \Omega_0 = 0$ and we obtain that $\Omega_{\text{eff}}(0) = \Omega_0$. Therefore, unlike previous couplers based on counterdiabatic STA [26], the proposed coupler does not require an increase in the coupling strength at maximum coupling.

### 3.1. An example

We plot the phase mismatch and the coupling as a function of the device length $z$ for the original phase mismatch model and
Figure 3. Light propagation simulation in an STA directional coupler. The coupling strength $\Omega_{\text{eff}}$ and phase mismatch $\Delta_{\text{eff}}$ are shown in figure 2(a). Subsequently, the corresponding distance between the two waveguides $d$ and their relative width differences are given in figure 2(c). Based on these geometrical parameters, we run the simulation of this device to demonstrate the light intensity transferring from input to output waveguide.

for the STA model (equation (7)) and show them in figure 2(a). We set the device length at 0.6 mm and $\Omega_0 = 1 \text{ mm}^{-1}$, and $\Delta_0 = 5 \text{ mm}^{-1}$. Subsequently, for figure 2(b), we assume that the input light is injected into the waveguide 1 and we plot the light intensity, $I_1 = |c_2(z)|^2$, of waveguide 2. We assume the same waveguides parameters, as in figure 2(a). It is clear that the light transfer for the STA system is much more effective, as compared to the original one.

The geometry of the waveguide coupler is determined by the coupling strength $\Omega_{\text{eff}}(z)$ and the phase mismatch $\Delta_{\text{eff}}(z)$ parameters. The separation distance $d$ between the two waveguides can be well fitted by the hyperbolic secant form of coupling strength $\Omega_{\text{eff}}(z)$, which the coupling strength has the exponential relationship with the separation distance [6–9]. In addition, the connection between the phase mismatch $\Delta_{\text{eff}}(z)$ and the difference between the widths of the two waveguides, $\delta_w = w_1 - w_2$, is given by a linear relation [22, 24, 25]. The engineering of a sign flip in the phase mismatch at the maximum coupling point can be realized by switching the materials of the two waveguides. This would realize a swap of the propagation constants of the two waveguides, $\beta_{20}(z = -0) \rightarrow \beta_{21}(z = +0)$, and thus, the desired sign flip in $\Delta_{\text{eff}}(z)$.

In this example, we choose the same material setting up as in [24]. The material setting up as following: 3 $\mu$m thick SiO$_2$ ($n = 1.46$) on a Si ($n = 3.48$) wafer is used for the bottom cladding layer, the core consists of a 2.4 $\mu$m layer of BCB ($n = 1.53$), and the upper cladding is epoxy ($n = 1.50$). With this configuration, we can easily obtain the exponential relationship (between coupling strength and separation distance) and linear relationship (between detuning and width difference of two waveguides) by fitting the simulation from the paper [24]. Based on these relationships, we can map the coupling strength $\Omega_{\text{eff}}$ and detuning $\Delta_{\text{eff}}$ in to the geometrical parameters, given by figure 2(c). Subsequently, we perform the simulation for this example to demonstrate our device performance in the figure 3, by established on the geometrical parameters (figure 2(c)). From the result of simulation, the light energy transfers from input to output waveguide, which is consistent with figure 2(b).

4. Performance and advantages

4.1. Shorter device length

To compare the performance of the proposed STA waveguide coupler to the original one, we show the contour plots of the light intensity at the end of the device, $I_3(L)$, for a waveguide coupler with a sign flip in the phase mismatch with STA (top frame) and without STA (bottom frame). The phase mismatch is set to $\Delta_0 = 1 \text{ mm}^{-1}$, while we vary the maximum coupling strength $\Omega_0$ and the device length $2L$.

4.2. Robustness against parameter fluctuations

We continue to show the superiority of the counterdiabatic STA waveguide coupler as compared to the analogous device without STA by examining the light intensity transfer to waveguide 2, $I_3(L)$, as a function of the maximum coupling strength $\Omega_0$ and the phase mismatch $\Delta_0$. The device length is fixed at $2L = 10\text{ mm}$, while $\Omega_0$ varies from $0 \text{ mm}^{-1}$ to $5 \text{ mm}^{-1}$ and $\Delta_0$ from $0 \text{ mm}^{-1}$ to $5 \text{ mm}^{-1}$, as shown in figure 5. We note that
the fidelity of the light transfer for the STA coupler is robust against variations in both the maximum coupling and phase mismatch, including around small $\Delta_0$, which is not the case for a light coupler without STA.

The stability of the light transfer efficiency to changes in the coupling and phase mismatch guarantees the achromatic operation of the proposed coupler. Consequently, varying wavelength of input light changes the propagation constants and mode profiles with against different input wavelength. Therefore, the fluctuations of wavelength modifies the coupling strength and detuning, based on CMT [6, 7, 33]. Owing to the fact that different wavelengths of light have different coupling and phase mismatch parameters, figure 5 clearly shows that these will not affect the fidelity of the light transfer within a moderate wavelength range.

To justify our design robustness against fabrication error, we introduce the random Gaussian noise into our coupling strength and detuning as amplitude of Gaussian noise divides the fidelity of the light transfer for the STA coupler. Consequently, uniform distribution random Gaussian noise is introduced into the coupling and phase mismatch in proportion to the maximum coupling error conduces gap error within 84 nm and 10% coupling strength error conduces gap error within 186 nm (at maximum coupling strength $\Omega_0 = 5 \text{ mm}^{-1}$), based on material structure [24].

The error rate of coupling strength and detuning by introducing random Gaussian noise (from 0% to 30%) against error rate of fidelity.

It is the reasonable to describe the fabrication error, because the coupling strength is depended on the distance between two waveguides [7, 8, 14] and detuning is linearly dependent with refraction indexes and difference between widths of two waveguides [22–24]. From this perspective, we effectively introduce the Gaussian noise to distance between two waveguides and difference of widths of two waveguides.

We run the 500 times to average the error rate of the fidelity, where the fidelity is the final transferring intensity of output waveguide $I_3(L)$ and error rate of the fidelity is $1 - I_3(L)$.

The error rate of coupling strength and detuning by introducing random Gaussian noise (figure 6), it is clearly showed that the error rates of coupling strength and detuning are as larger as 30%, the fidelity error rate is relatively small (within 15%) and also our shortcut method has much smaller error rate than the original phase mismatch model. In addition, according to the exponential (coupling strength and distance) and linear (detuning and width difference) relationships, it is easy to estimate that 10% detuning error leads to width difference error within 84 nm and 10% coupling strength error conduces gap error within 186 nm (at maximum coupling strength $\Omega_0 = 5 \text{ mm}^{-1}$), based on material structure [24].

5. Beam splitter based on STA

In this section, we consider three coupled waveguides as shown in figure 6. We assume that the outer waveguides, waveguides 1 and 3, are geometrically symmetric with respect to waveguide 2, that is they are equally coupled to it with $\Omega(z)$. Furthermore, the outer waveguides are assumed to have equal refractive indexes $n_2$, while the refractive index of the middle waveguide changes from $n_1$ to $n_3$ at the maximum of the coupling, $z = 0$. We assume that waveguide 2 in the center that has constant width and input/output waveguide has the same width variation, such that $w_1(z) = w_3(z)$. The phase mismatch is defined as $\Delta(z) = \beta_2(z) - \beta_3(z)$, where $\beta_1(z)$ and $\beta_2(z)$ are the propagation coefficients of waveguides 1 and 2. The light propagation in this waveguide array is described by

$$i \frac{d}{dz} \begin{bmatrix} c_1(z) \\ c_2(z) \\ c_3(z) \end{bmatrix} = \begin{bmatrix} 0 & \Omega(z) & 0 \\ \Omega(z) & \Delta(z) & \Omega(z) \\ 0 & \Omega(z) & 0 \end{bmatrix} \begin{bmatrix} c_1(z) \\ c_2(z) \\ c_3(z) \end{bmatrix}. \quad (8)$$

These coupled differential equations are analogous to the Schrödinger equation describing a three-state quantum system subjected to an external electromagnetic field. Thus, we can introduce a new basis of a dark $c_d(z) = \frac{1}{\sqrt{2}}(c_1(z) - c_3(z))$ and a bright state $c_b(z) = \frac{1}{\sqrt{2}}(c_1(z) - c_3(z))$. Rewriting equation (8) in the new basis,

$$i \frac{d}{dz} \begin{bmatrix} c_b(z) \\ c_d(z) \end{bmatrix} = \begin{bmatrix} 0 & \sqrt{2}\Omega(z) \\ \sqrt{2}\Omega(z) & \Delta(z) \end{bmatrix} \begin{bmatrix} c_b(z) \\ c_d(z) \end{bmatrix}, \quad (9)$$

The phase mismatch model. In addition, according to the exponential (coupling strength and distance) and linear (detuning and width difference) relationships, it is easy to estimate that 10% detuning error leads to width difference error within 84 nm and 10% coupling strength error conduces gap error within 186 nm (at maximum coupling strength $\Omega_0 = 5 \text{ mm}^{-1}$), based on material structure [24].

5. Beam splitter based on STA

In this section, we consider three coupled waveguides as shown in figure 6. We assume that the outer waveguides, waveguides 1 and 3, are geometrically symmetric with respect to waveguide 2, that is they are equally coupled to it with $\Omega(z)$. Furthermore, the outer waveguides are assumed to have equal refractive indexes $n_2$, while the refractive index of the middle waveguide changes from $n_1$ to $n_3$ at the maximum of the coupling, $z = 0$. We assume that waveguide 2 in the center that has constant width and input/output waveguide has the same width variation, such that $w_1(z) = w_3(z)$. The phase mismatch is defined as $\Delta(z) = \beta_2(z) - \beta_3(z)$, where $\beta_1(z)$ and $\beta_2(z)$ are the propagation coefficients of waveguides 1 and 2. The light propagation in this waveguide array is described by

$$i \frac{d}{dz} \begin{bmatrix} c_1(z) \\ c_2(z) \\ c_3(z) \end{bmatrix} = \begin{bmatrix} 0 & \Omega(z) & 0 \\ \Omega(z) & \Delta(z) & \Omega(z) \\ 0 & \Omega(z) & 0 \end{bmatrix} \begin{bmatrix} c_1(z) \\ c_2(z) \\ c_3(z) \end{bmatrix}. \quad (8)$$

These coupled differential equations are analogous to the Schrödinger equation describing a three-state quantum system subjected to an external electromagnetic field. Thus, we can introduce a new basis of a dark $c_d(z) = \frac{1}{\sqrt{2}}(c_1(z) - c_3(z))$ and a bright state $c_b(z) = \frac{1}{\sqrt{2}}(c_1(z) - c_3(z))$. Rewriting equation (8) in the new basis,

$$i \frac{d}{dz} \begin{bmatrix} c_b(z) \\ c_d(z) \end{bmatrix} = \begin{bmatrix} 0 & \sqrt{2}\Omega(z) \\ \sqrt{2}\Omega(z) & \Delta(z) \end{bmatrix} \begin{bmatrix} c_b(z) \\ c_d(z) \end{bmatrix}, \quad (9)$$

The phase mismatch model. In addition, according to the exponential (coupling strength and distance) and linear (detuning and width difference) relationships, it is easy to estimate that 10% detuning error leads to width difference error within 84 nm and 10% coupling strength error conduces gap error within 186 nm (at maximum coupling strength $\Omega_0 = 5 \text{ mm}^{-1}$), based on material structure [24].
we can easily see that the dark state \( c_d(z) \) is decoupled, and the three-state problem is reduced to a two-state one involving states \( c_b(z) \) and \( c_c(z) \) only.

This set of differential equations is the same as the one for two coupled waveguides. Therefore, if we use the same parameters with STA \( \Omega(z) \) and \( \Delta(z) \) from equation (7), we can realize a robust and fast complete state transfer from \( c_d(z) \) to \( c_c(z) \). Mapping to the three coupled waveguides system, if we assume that light is initially input in the middle waveguide, then the final output light will be in state \( c_d(z) \), which realizes an equal intensity superposition between the two outer waveguides. With an example to demonstrate the performance of our beam splitter, we set up the parameters as \( \Delta_0 = 1 \text{ mm}^{-1} \), \( \Omega_0 = 5 \text{ mm}^{-1} \) and device length \( 2L = 0.6 \text{ mm} \). The light intensity evolution of waveguides \( I_1, I_2 \) and \( I_3 \) are shown in figure 7(b) and it is easy to obtain that our shortcut of phase mismatch model has better performance than original phase mismatch model. Note that by design, this device will have the same advantages as the two-waveguide coupler, these are (i) a shorter device length; (ii) achromaticity; and (iii) robustness to parameter fluctuations including around small values for the phase mismatch.

Employing phase mismatch model in optical waveguide achieves achromaticity and robustness against geometrical parameters device which is proposed in [30]. In this paper, we further improve the performance of phase mismatch model device, such as the shorter device length (figure 4) and improved robustness (figure 5). Indeed, physical implementation of sudden change for phase mismatch is an engineering problem. However, we can employ the ion implementation technique [15] by changing different concentration of ions to vary the refraction indexes. Another method, which is given by our example in our paper, is using the same material structure in [24]. We can implement the required sudden change for phase mismatch by changing the width difference to vary the detuning.

6. Conclusions

We demonstrated a novel device for complete achromatic optical switching between evanescently coupled waveguides. Utilizing counterdiabatic STA-obtained changes to the coupling and the phase mismatch, we show that the proposed device realizes a complete and robust achromatic light switching on a shorter length scale as compared to previous designs. We note that the required parameter changes obey the coupling strength inequality, \( |\Delta_0(z)| \leq |\Omega(z)| \leq |\Omega_0(z)| \). Finally, we showed that a similar waveguide coupler with STA can be used for realizing an equal superposition beam splitter in a system of three coupled waveguides.

Acknowledgments

W H acknowledges funding from the National Science and Technology Major Project (Grant No. 2017ZX02101007-003); National Natural Science Foundation of China (Grant No. 61565004) and National Natural Science Foundation of China (Grant No. 61665001).

L K A is partly supported by Singapore ASTAR AME IRG A1783c0011 and US Air Force Office of Scientific Research (AFOSR) through the Asian Office of Aerospace Research and Development (AOARD) under Grant No. FA2386-17-1-4020.

E K acknowledges financial support from the European Union’s Horizon 2020 research and innovation programme under the Marie Sklodowska-Curie Grant agreement No. 705256—COPQE.

ORCID iDs

Wei Huang https://orcid.org/0000-0002-1072-522X
Ellica Kyoseva https://orcid.org/0000-0002-9154-0293

References

[29] Chen X et al 2018 Compact beam splitters in coupled waveguides using shortcuts to adiabaticity J. Opt. 20 045804