

Theory of shot noise in high-current space-charge-limited field emission

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This paper studies the electron shot noise of high current space-charge limited (SCL) field emission, including the effects of both quantum partitioning and Coulomb correlation of the emitted tunneling electrons inside a gap. The Fano factor (γ) is calculated over a wide range of applied voltages, gap spacings, and electron pulse lengths in classical, quantum and relativistic regimes. It is found that the effect of space charge smoothing (observed in thermionic emission) is no longer valid for high current field emission, as the space charge field will increase the potential barrier and thus decrease the degree of quantum partitioning (larger γ). However, the shot noise of SCL field emission in a nanogap may be suppressed due to the combination of Coulomb repulsion and exchange-correlation potential among the tunneling electrons, where the dynamics of the electrons inside the gap are treated by using a quantum mean field model. Coulomb correlation can be ignored when the pulse length of the electron beam is much smaller than the gap transit time. Possible experimental settings are discussed in order to observe the shot noise suppression of field emission with and without the space charge effects.

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Shot noise is a type of electronic noise that occurs due to the discreteness of electron charges. Its spectral power density is linearly proportional to the current and independent of the frequency. By considering the electron emission from a thermionic cathode (in a vacuum tube) as a Poisson process, Schottky¹ first proposed that the spectral power density of shot noise (at low frequency) is $S=2eI$, where e is electron charge and I is the mean value of the transmitted current. Recent studies of shot noise have been focused on mesoscopic devices,² which can provide crucial information that is not available in the time-averaged value, such as the particle-wave duality of electron transport.

The deviation from the uncorrelated shot noise is commonly termed as the Fano factor $\gamma=S/2eI$, where $\gamma<1$ indicates the suppression of shot noise due to the correlations among the electrons. It was firstly found that the shot noise could be smoothed out by the space charge effects (or Coulomb repulsion among the electrons) for thermionic emission operating at space-charge limited (SCL) condition.³ For mesoscopic devices, the main source of correlation is the Pauli exclusion principle, which provides the effect of quantum partitioning for shot noise suppression observed in many systems, such as ballistic quantum point contact,^{4,5} nondegenerate diffusive conductors,^{6,7} chaotic system,⁸ and avalanche photodiodes.⁹ Suppression of shot noise due to Coulomb correlation was also studied in ballistic conductors.¹⁰

In a recent paper,¹¹ the shot noise from electron field emission was analyzed as a quantum scattering problem within the Landauer-Büttiker framework. They indicated that quantum suppression of shot noise is a result of quantum partitioning between the transmitted electrons through, and the reflected electrons from the potential barrier near the cathode-vacuum interface. The study was later extended to a more realistic potential barrier profile with an image force potential using a quantum-mechanical wave impedance approach, and analytical expressions for the Fano factor were derived for both pure field emission (Fowler-Nordheim law) and pure thermionic emission (Richardson law).¹² However,

the space charge effects (or Coulomb correlation) of the tunneling electrons were ignored completely, which would become important for high current field emission. Such high current field emission are relevant in the fabrication of large area SCL carbon nanotube field emitters,¹³ the applications for high frequency high power microwave sources,¹⁴ and short-pulse field emission using ultrafast laser excitation.¹⁵

Depending on the gap spacing, applied voltage, and electron pulse width, the Coulomb interaction and the dynamics of electrons have to be treated quantum mechanically when the electron de Broglie wavelength is comparable to the characteristic length scales^{16,17} or relativistically at high voltages.¹⁷ Therefore, in this context, several interesting questions arise: What is the suppression of shot noise for SCL field emission operating over a wide range of parameters? What is the role of Coulomb correlation in the presence of quantum partitioning? Can we have the effect of space-charge smoothing in field emission similar to thermionic emission?

Following the Landauer-Büttiker framework, the Fano factor of the field emission from a metallic surface can be expressed as^{11,12}

$$\gamma = 1 - \frac{\int_{-\infty}^{+\infty} C_T^2(E_x) g[-\beta E_x] dE_x}{\int_{-\infty}^{+\infty} C_T(E_x) h[-\beta E_x] dE_x}, \quad (1)$$

where $\beta=1/k_B T$, k_B is Boltzmann's constant, T is temperature, E_x is the kinetic energy in the current flow direction (with respect to the Fermi level E_F), $C_T(E_x)$ is the transmission coefficient through the potential energy barrier, $h(z)$ and $g(z)$ are defined as $h(z)=\ln(1+e^z)$, and $g(z)=h(z)-dh(z)/dz$.

The transmitted electron current density J is obtained by solving

$$J = e \int_{-\infty}^{\infty} N(E_x) C_T(E_x) dE_x, \quad (2)$$

where $N(E_x) = (mk_B T / 2\pi^2 \hbar^3) \ln[1 + \exp(-E_x/k_B T)]$ is the density of incident electrons. The transmission coefficient $C_T(E_x)$ is calculated using Miller–Good approximation:¹⁸ $C_T = \{1 + \exp[\theta(E_x)]\}^{-1}$, where

$$\theta(E_x) = \frac{2}{\hbar} \int_{x_1}^{x_2} \sqrt{2m[eV(x) - E_x]} dx.$$

To calculate C_T , we consider that the electrons are emitted from the cathode at $x=0$, with an applied anode voltage V_g at $x=D$. The electrical potential across the gap is

$$V(x) = \Phi_{WF} + \Phi_{IM} - \Phi_V - \Phi_{SC} + \Phi_{XC}, \quad (3)$$

where Φ_{WF} is the work function of the field emitter, $\Phi_{IM} < 0$ is the image charge potential, $\Phi_V = Fx$ is the external applied electrical potential with $F = V_g/D$, and $\Phi_{XC} < 0$ and $\Phi_{SC} < 0$ is, respectively, the exchange-correlation and the space-charge potential due to transmitted tunneling electrons in the gap, which are calculated self-consistently (see below). To include the effect of anode screening, we first assume the image charge term is

$$\Phi_{IM}(x) = - \left[\frac{e}{16\pi\epsilon_0(x+x_0)} + \frac{e}{8\pi\epsilon_0} \sum_{n=1}^{\infty} \left(\frac{nD}{n^2 D^2 - x^2} - \frac{1}{nD} \right) \right], \quad (4)$$

where x_0 is a value selected so that the image charge potential profile is continuous across the cathode-vacuum interface according to $x_0 = e/[16\pi\epsilon_0(E_F + \Phi_{WF})]$.¹² The second term represents the anode image charge correction, which will be significant for field emission in a sub-10 nm gap. A more accurate formulation of the image charge potential based on a Thomas–Fermi approximated quantum model will also be used to include the spillover effect of the electrons (from both electrodes into the vacuum) and also to remove the singularities at the electrodes (see below).

To calculate Φ_{SC} and Φ_{XC} for an emitting current density J , we use the Schrödinger–Poisson approach and the relativistic formulation of the Poisson equation, respectively, to account for the quantum dynamics (for a nanogap) and relativistic effects (for large gap voltage). The normalized time-independent Schrödinger and Poisson equations in a quantum mean field model¹⁶ are

$$q'' + \lambda^2 \left[\phi_{sc} + \bar{x} - \phi_{xc} - \frac{4\mu^2}{9q^4} \right] q = 0, \quad (5)$$

$$\phi_{sc}'' = \frac{2}{3} q^2, \quad (6)$$

where q is the normalized wave amplitude, $\phi_{sc} = \Phi_{SC}/V_g$ is the normalized space-charge potential, $\phi_{xc} = \Phi_{XC}/V_g$ is the normalized exchange-correlation potential based on the local density functional theory,¹⁹ q'' (ϕ_{sc}'') denote the second derivative of q (ϕ_{sc}) with respect to $\bar{x} = x/D$, $\lambda = D/\lambda_o$ is the normalized gap spacing with respect to the electron de Bro-

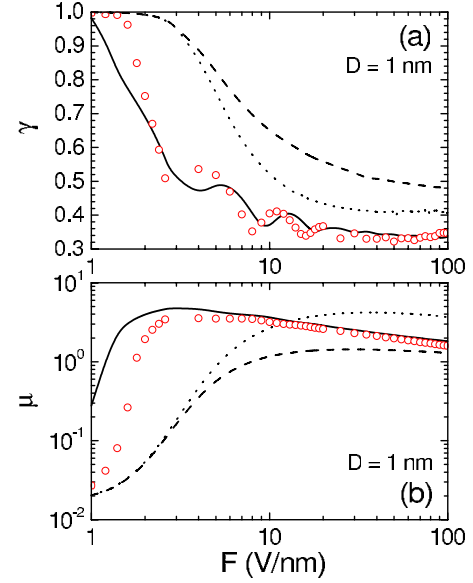


FIG. 1. (Color online) (a) The Fano factor γ and (b) the normalized current density μ at $x_0=0$, $T=300$ K, and $\Phi_{WF}=2.48$ eV in a quantum gap ($D=1$ nm) due to the Coulomb correlation as a function of F (solid lines). The dashed lines are the calculations without the exchange-correlation potential ($\Phi_{XC}=0$). The dotted lines are the calculations without the Coulomb correlation ($\Phi_{XC}=\Phi_{SC}=0$). The symbols represent the calculations of finite $x_0=e/[16\pi\epsilon_0(E_F + \Phi_{WF})]$.

glie wavelength at V_g : $\lambda_o = \hbar/\sqrt{2emV_g}$, and $\mu = J/J_{CL}$ is the normalized electron current density with respect to the classical Child-Langmuir (CL) current density,²⁰

$$J_{CL} = \frac{4\epsilon_0}{9} \sqrt{\frac{2e}{m}} \frac{V_g^{3/2}}{D^2}. \quad (7)$$

The Poisson equation in the relativistic regime is^{21,17}

$$\Gamma'' = \mu \frac{4\sqrt{2}}{9} U_o^{3/2} \frac{\Gamma}{\sqrt{(\Gamma^2 - 1)}}, \quad (8)$$

where $\Gamma = 1 + e(\Phi_V + \Phi_{SC})/(mc^2)$ and $U_o = eV_g/(mc^2)$. At $\lambda \gg 1$ or $U_o \ll 1$, ϕ_{xc} is negligible, and the space charge field ϕ_{sc} will recover the classical limit by solving

$$\phi_{sc}'' = \frac{4}{9} \frac{\mu}{\sqrt{\phi_{sc} + \bar{x}}}. \quad (9)$$

Hence, for a given work function Φ_{WF} and temperature T , we can calculate the Fano factor γ and normalized current density μ by solving the above equations numerically, over a wide range of gap spacings and voltages. In particular, we will study three systems with different D and V_g in a range of electric field strength from $F=1$ to 100 V/nm: quantum gap at $D=1$ nm, classical gap at $D=1$ μm , and relativistic gap at $D=0.1$ mm. Unless it is specified elsewhere, we set $\Phi_{WF}=2.48$ eV (Ba material) and $T=300$ K in all our calculations.

In Fig. 1, the calculated (a) γ and (b) μ are plotted as a function of F at $x_0=0$. For comparison, the calculations without including the Coulomb correlation ($\Phi_{SC}=0$ and $\Phi_{XC}=0$)

are plotted in dotted lines, similar to the previous calculations.^{11,12} It is clear that the Coulomb correlation among the tunneling electrons will further reduce the shot noise. However, this further suppression of the shot noise is due to the *combination* of Coulomb correlation by the space charge field (Coulomb repulsion) and the exchange-correlation effects of the tunneling electrons.

To illustrate this argument, we calculate a case with only the space charge field by ignoring the exchange-correlation effects ($\Phi_{XC}=0$) and plot it using dashed lines. In Fig. 1(a), it shows a larger γ as compared to pure field emission (dotted lines). The finding suggests that pure space charge field or Coulomb repulsion of the emitted electrons will decrease the effect of quantum partitioning by creating a larger potential barrier. This leads to smaller quantum suppression and larger γ . On the other hand, the exchange-correlation effects of transmitted electrons will result in a smaller potential barrier, which enhances the electron tunneling (larger current), as shown in Fig. 1(b), and the suppression of shot noise (smaller γ), as shown in Fig. 1(a). Note this quantum Coulomb correlation is due to the quantum dynamics of high current electron flows in a nanogap, as the SCL current density (in quantum regime) has exceeded the classical CL law ($\mu > 1$).¹⁶

All the calculations above have assumed a classical image charge potential term with $x_0=0$. A case with nonzero $x_0 = e/[16\pi\epsilon_0(E_F + \Phi_{WF})]$ is also plotted in Fig. 1 (symbols) for comparison. The effect of image charge potential ($x_0 \neq 0$) is only important at small $F < 3$ V/nm, and the space charge effects will become dominant at higher F and J (or μ). A similar finding is also valid when a more accurate Thomas-Fermi approximation (TFA) based image charge potential is used later, as shown in Fig. 5 below.

The degree of quantum suppression of SCL field emission including the effects of both Φ_{XC} and Φ_{SC} will depend on the gap spacing, work function, and temperature. Figure 2 shows the Fano factor γ (solid) and SCL current density μ (dashed) as functions of F for (a) $D=1, 10,$ and 1000 nm, (b) $\Phi_{WF}=2.14, 2.48, 3.76,$ and 4.4 eV, and (c) $T=300, 1000,$ and 2000 K. For a fixed electric field F , γ decreases with a smaller gap spacing D because of the manifest of quantum Coulomb correlation at smaller D , as shown in Fig. 2(a). Similarly in Fig. 2(b), γ will decrease with smaller work function, as the effects of quantum partitioning become more significant with a smaller potential barrier. This noise suppression effect originates from the Fermi statistical correlations under the condition of current partitioning since the noise power is determined by the relative position of the Fermi level with respect to the barrier height.

At high current density, it is expected that the resistive heating on the field emitter will increase the temperature of the tip. For this practical issue, we study the effects of high temperature (up to 2000 K), as shown in Fig. 2(c). From the figure, we see that γ will increase with temperature at high electric field ($F > 3$ V/nm), for which the regime of SCL field emission has been reached (same μ at different T). At a relatively low $F=1$ V/nm and $T=2000$ K, the emission behaves like a thermionic enhanced field emission, which shows a slight suppression of shot noise with $\gamma=0.92$. This suppression is due to the effect of space charge smoothing, as

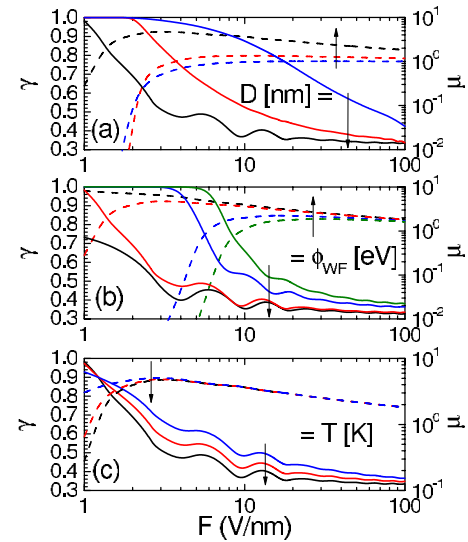


FIG. 2. (Color online) The Fano factor γ (solid) and the normalized current density μ (dashed) in a quantum gap (with $x_0=0$) due to the Coulomb correlation for (a) $D=1, 10,$ and 1000 nm, (b) $\Phi_{WF}=2.14, 2.48, 3.76,$ and 4.4 eV, and (c) $T=300, 1000,$ and 2000 K. The arrows indicate the direction of decreasing D , Φ_{WF} , and T .

expected in the thermionic-like emission, which is dominant at low field and high temperature.

In Fig. 3(a), we show the calculated results in a classical gap at $D=1$ μm for $F=1$ to 100 V/nm ($V_g=1$ to 100 kV) at

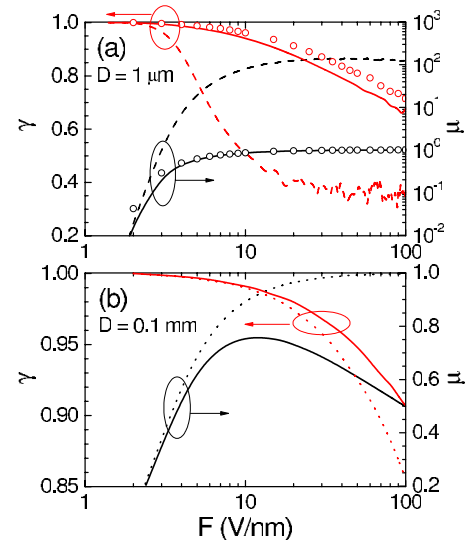


FIG. 3. (Color online) The Fano factor γ (left label) and the normalized current density μ (right label) at $T=300$ K and $\Phi_{WF}=2.48$ eV due to the Coulomb correlation of the space-charge field (Coulomb repulsion) for $D=(a)$ 1 μm (classical) and (b) 0.1 mm (relativistic). The dashed lines in (a) are the calculation of pure field emission. The dotted lines in (b) are the calculation without the relativistic effects. The symbols in (a) are the calculation at $T=1000$ K.

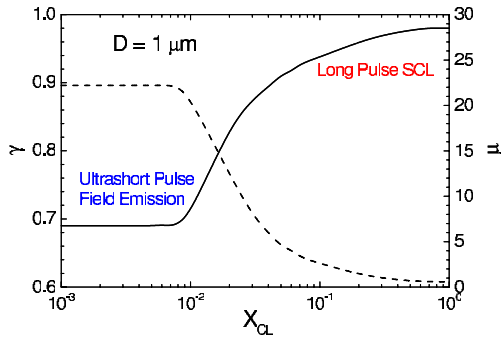


FIG. 4. (Color online) The Fano factor γ (solid) and the normalized current density μ (dashed) at $T=300$ K, $\Phi_{WF}=2.48$ eV, $D=1$ μm , and $F=5$ V/nm due to the Coulomb correlation of the space charge field as a function normalized electron pulse length X_{CL} in terms of the gap transit time.

$T=300$ K. In the classical regime of large gap spacing, the exchange-correlation effect of emitting electrons inside the gap is negligible on the dynamics of the electron transport. The Coulomb correlation is mainly due to the classical Coulomb repulsion of the space charge field, which will reduce the shot noise suppression (solid lines) and thus produce a large γ as compared to pure field emission (dashed lines), as shown in Fig. 3(a). This finding implies that there is no space-charge smoothing for space-charge limited field emission operating for a classical gap at high F . In this case, the effective suppression of shot noise for the SCL field emission is due to the saturation of the field emission current due to space charge effects ($\mu=1$), but with a larger value of Fano factor ($\gamma>0.6$) as compared to the pure field emission without space-charge effects. A calculation at higher temperature $T=1000$ K is also plotted (symbols) for comparison in Fig. 3(a). As expected, high temperature will not change the SCL current density ($\mu=1$), but increase the Fano factor at high electric field.

In Fig. 3(b), we compare the results with (solid) and without (dotted) including the relativistic effects at $D=0.1$ nm for $F=1$ to 100 V/nm ($V_g=0.1$ –10 MV). It is found that the relativistic effect will produce less shot noise suppression, but the difference is not significant: $\gamma=0.91$ (relativistic) and 0.86 (classical) at $F=100$ V/nm. The difference of γ is negligible at low $F<10$ V/nm.

Due to the advances in femtosecond lasers, it is now possible to have electron emission with a very short pulse length through the assisted field emission by optical electric field.¹⁵ A model of such ultrafast electron bunches operating at SCL condition has been studied recently,¹⁷ where the pulse length (τ) of the electron beam is less than the gap transit time at SCL condition: $T_{CL}=3D\sqrt{m/2eV_g}$. Here, we study the effects of electron pulse length on the shot noise suppression by including the space charge field in an equivalent diode model.¹⁷ The details of the formulation will be published elsewhere. In Fig. 4, we show the calculated Fano factor γ (solid) and normalized current density μ (dashed) as functions of normalized pulse length $X_{CL}=\tau/T_{CL}$ at fixed $F=5$ V/nm for a large gap of $D=1$ μm ($V_g=5$ kV). At short pulse length, $X_{CL}\ll 1$, the SCL current density is very high as

it is inversely proportional to X_{CL} .¹⁷ Thus, the field emission is basically source limited where the space-charge effect is not important, and γ is not a function of X_{CL} , which is about $\gamma=0.7$, similar to pure field emission case calculated before [see the dashed line in Fig. 3(a) at $F=5$ V/nm]. At large X_{CL} approaching the long pulse (steady-state) limit at $X_{CL}=1$, the space-charge effect is significant to increase the potential barrier, leading to smaller quantum partitioning and larger $\gamma=0.98$ [see also the solid line in Fig. 3(a) at $F=5$ V/nm]. Thus, the Coulomb correlation can be ignored for the suppression of shot noise, if the pulse length is short enough for source-limited electron emission, where space charge effect is negligible.

It is clear that the classical image charge potential shown in Eq. (4) is not valid to account for the spillover effects of the electrons into the vacuum region near to the electrodes, especially for a gap less than 10 nm. To improve on this limitation such as removing the singularities near to the electrodes, we solve a TFA model to obtain the ground state (and zero field) image-charge potential Φ_{IM} , which is due to the Coulomb repulsion of the charges in the cathode, vacuum, and anode. The TFA-based image charge potential is

$$\Phi_{IM} = -e \int_0^\infty p dp \left[D_{vac}(p, x) + \frac{1}{2p} \right], \quad (10)$$

where the $D_{vac}(p, x)$ term is the longitudinal self-consistent field describing the screened Coulomb interaction between two charges at position x and x' , and p is the wave vector along the x direction. The formulation is basically by solving the screened Poisson equation in its Green's function form by using a TFA method.²² The details are not discussed here, and will be published elsewhere.²³ Note that this TFA approach can be considered as a type of density-gradient theory, which has been used in the analysis of field emission from metals.²⁴

In Fig. 5(a), it is shown that the (ii) TFA image charge potential is continuous at both metal-vacuum interfaces for Barium (Ba) electrodes at $D=1$ nm. For the (i) classical image-charge potential with the effect of anode screening given by Eq. (4) at $x_0=0$, the singularities of Φ_{IM} at the metal-vacuum interfaces may cause an underestimation of the barrier width. In Figs. 5(b) and 5(c), we compare, respectively, the calculated γ and μ for $F=1$ to 10 V/nm for the two different forms of image charge potentials. By comparing the solid lines of (i) and (ii) in Fig. 5(b), we see that the influence of the image charge potential on the Fano factor is more significant at low field $F<3$ V/nm. At relatively high field $F>5$ V/nm, the Fano factor varies between $\gamma=0.37$ and 0.42 and the field emission current saturates at the space charge limited condition in the quantum regime with a value of $\mu=2$ –3, as shown in Fig. 5(c). For completeness, the calculations without including the Coulomb correlation ($\Phi_{SC}=0$ and $\Phi_{XC}=0$) are plotted in dotted lines, and the calculations with only the space-charge field but no the exchange-correlation effects ($\Phi_{XC}=0$) are plotted using dashed lines. The finding on using the TFA approach is similar to the ones using the classical image potential shown in Fig. 1, where the combination of Coulomb repulsion by the

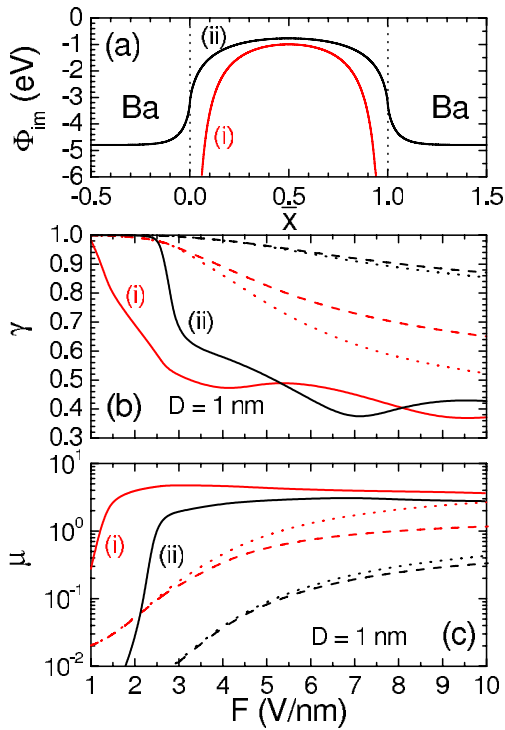


FIG. 5. (Color online) (a) The image charge potential profile Φ_{IM} for (i) the classical Ba-vacuum-Ba system in Eq. (4) and (ii) the TFA quantum Ba-vacuum-Ba system for a quantum gap ($D = 1$ nm) of barium (Ba) electrodes ($E_F = 2.317$ eV and $\Phi_{WF} = 2.48$ eV). (b) The Fano factor γ and (c) the normalized current density μ at $T = 300$ K due to the Coulomb correlation as a function of F (solid lines) for the two different forms of image charge potentials. The dashed lines are the calculations without the exchange-correlation potential ($\Phi_{XC} = 0$). The dotted lines are the calculations without the Coulomb correlation ($\Phi_{XC} = \Phi_{SC} = 0$).

space-charge field and the exchange-correlation effects of the tunneling electrons inside the nanogap will reduce the shot noise with a smaller Fano factor γ . The Coulomb repulsion (dashed lines) alone will increase the Fano factor as compared to pure field emission (dashed lines).

To the best of our knowledge, there is no experimental measurement of shot noise reduction for field emission even at low current regime (without space charge effects). Most studies of the electron emission noise have been focused on the flicker noise at a very low frequency^{25,26} with a spectrum proportional to $1/f^b$, where $b=1$ for CNT cathode²⁵ and $b=3/2$ for ZnO cathode.²⁶ The main difficulty in measuring shot noise is the requirement of high frequency and dedicated setup in order to avoid the significance of flicker noise at low frequency and to distinguish the relatively small shot noise spectrum from the background noise.

Based on the flicker noise data (up to 25 Hz) from a recent field emission experiment on a single CNT operating at a local field of about 6 V/nm and 600 K, with an emitting current of 2.4 nA,²⁵ we estimate the frequency range to reach the shot noise regime. The experiment gives a normalized noise spectrum of $S/I^2 = a/f$, where $a = 4.5 \times 10^{-9}$ and f is the frequency in Hertz. The CNT has a work function of 5 eV according to their experimental Fowler–Nordheim plotting.

At this operating condition (no space-charge effects), our calculation shows no shot noise suppression and the calculated Fano factor is about 1 ($\gamma = 0.9999$). By setting the flicker noise equal to the classical full shot noise, the minimum frequency that the shot noise will become dominant is at $f > 35$ Hz, where the normalized full shot noise is about $S/I^2 = 2e/I = 1.3 \times 10^{-10}$ Hz⁻¹. From the results at its maximum measurable frequency of 25 Hz, the measured normalized noise spectrum has a large variation from 0.8 to 3×10^{-10} Hz⁻¹ due to the uncertainty in the measurement system. Thus, it is required to have a low work function material (< 3 eV), operating at a reasonable electric field of < 10 V/nm in order to observe the significant shot noise suppression. By using the same conditions mentioned above but with a smaller work function of 3 eV, the Fano factor is about 0.88, according to our calculation with no space-charge effects. To observe the shot noise of space-charge limited field emission studied in this paper, especially in a nanogap, the work function has to be even smaller, such as lanthanum sulfide (LaS) material, which has a work function of about 1.14 eV at 300 K.¹²

It is important to note the model proposed here is an equilibrium model, where the electronic properties of the metal surfaces are assumed to be at equilibrium. To include the nonequilibrium behavior of the electrons, nonequilibrium Kohn–Sham theory or Green’s function approaches are required, which is beyond the scope of this paper. Here, we would like to make some comments on the consistency of our model. First, our equilibrium model is able to show a reduction of shot noise for thermionic enhanced field emission, as shown in Fig. 2(c) at $F = 1$ V/nm and $T = 2000$ K. This finding is consistent with the known effect of space charge smoothing for thermionic emission. Using the TFA based model including the effects of Coulomb repulsion and exchange correlation of the tunneling transmitted electrons, the model gives a better estimation on the emission area of an experimental IV curve in a nanogap.²⁷ Our model²³ estimates an area of about 1.2 nm² for a $D = 0.8$ nm, which is more consistent than the value of 0.004 nm² calculated based on a classical model.²⁸ Finally, our model is able to show a smooth transition to the classical SCL current (or classical CL law) with $\mu = 1$ at a large gap spacing for which the quantum effects are expected to be insignificant.¹⁶

In conclusion, the influence of the Coulomb correlation on the degree of quantum partitioning has been studied for the SCL field emission by including the space charge field and exchange-correlation potential of the tunneling electron inside a gap. The Fano factor is calculated over a wide range of gap spacings, applied voltages, and pulse lengths in classical, quantum, and relativistic regimes. For field emission in a large gap, Coulomb correlation will increase the Fano factor (smaller suppression of shot noise) because the space-charge field will increase the potential barrier (less quantum partitioning). Therefore, there is no effect of space-charge smoothing in SCL field emission, different from what is predicted in SCL thermionic emission. Nevertheless, in a nanogap with low voltage, the combination of space charge field and exchange-correlation potential of the tunneling electrons

will enhance the suppression of shot noise as compared to pure field emission. At short pulse limit (smaller than gap transit time), space-charge effect can be ignored, and thus, a larger suppression of shot noise is predicted for short electron pulses.

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