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Electromagnetic sinc Schell-model pulses in dispersive media

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A class of random electromagnetic pulsed sources with sinc Schell-model correlations is introduced. Analytical formulas for the electromagnetic pulses generated by such pulsed sources propagating in dispersive media are derived. It is shown that the temporal intensity distribution of this new type of pulse exhibits unique propagation features, such as reshaping its average intensity from the initial Gaussian profile to a double-layer flat-top distribution at far field. The effects, arising from the source temporal coherent length and the dispersion coefficient, on the profiles of the temporal intensity distribution and the temporal degree of polarization are analyzed in detail. The results presented here demonstrate the potential of coherence modulation for pulse shaping applications.

1. Introduction

The evolution of stationary random fields in free space and in media has been extensively studied on the basis of the general theory developed in [1,2]. It is well known that the shape of the degree of coherence in the source plane is intimately related to its far-field intensity distribution, and these two quantities are related by a Fourier-type law. Most of the studies concerning the correlation-induced effects are confined to the traditional Gaussian-Schell model (GSM) sources, for which the degree of coherence between two spatial points is assumed to depend only on their separation. The sufficient condition for devising a genuine correlation functions of partially coherent sources was established by Gori et al. [3,4]. In addition to the well-explored GSM sources, different special model based sources have been recently introduced including non-uniformly correlated (NUC) sources [5,6], Multi-Gaussian Schell-model (MGSM) sources [7,8], cosine-Gaussian Schell-model sources [9] and sinc Schell-model (SSM) sources [10,11] and others. It has been revealed that beams generated by these sources exhibit distinctive propagating characteristics.

While all the aforementioned literatures were carried out with respect to stationary fields, the non-stationary light fields, which exhibit partial coherence temporally or spectrally, have attracted much attention due to their crucial role in optical telecommunications [12]. In the past 10 years, the influence of the correlation-induced propagation effects of partially coherent pulses in various media has been studied both in time and frequency domains [13–21]. Recently several random pulse classes with correlation distribution of non-Gaussian Schell-model types were shown to exhibit interesting and useful features on propagation in dispersive media [22–26]. For instance, non-uniform correlated pulse produces a temporal shift in their peak intensity [22,23], cosine-Gaussian Schell-model pulse splits from a single average pulse distribution into two symmetric parts [24], and a partially coherent pulsed source with sinc Schell-model correlations can generate a flat-top intensity profile [25], somewhat analogous to the Multi-Gaussian Schell-model pulse [26]. The possibility of the experimental realization of the random pulses has been illustrated in [17].

In this paper we extend the scalar model [25] to the full electromagnetic domain, terming the novel class of pulses the *Electromagnetic sinc-Schell model* (EM SSM) pulses. In our model, we focus on the evolution behavior of the second-order statistical properties of EM SSM pulses in dispersive media. The effects of the temporal coherence length of incident pulses and the dispersion dispersive coefficient of medium on the profiles of average intensity and polarization are also emphasized.

2. Theoretical formulation

In the space-time domain, the second-order statistical properties of electromagnetic random pulses can be described by a 2×2 mutual coherence matrix $\overleftrightarrow{\Gamma}^{(0)}(t_1', t_2')$, whose element $\Gamma_{ij}^{(0)}(t_1', t_2')$ is defined as [1]

$$\Gamma_{ij}^{(0)}(t_1', t_2') = \langle E_i^*(t_1') E_j(t_2') \rangle, \quad (i = x, y; j = x, y). \tag{1}$$

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Here, $E_i(t')$ represents the electric field component along the i axis at time t' and the angular brackets stand for ensemble average. In order to be physically realizable, $\Gamma_{ij}^{(0)}(t_1', t_2')$ must have an integral representation in the form of [4]

$$\Gamma_{ij}^{(0)}(t_1', t_2') = \int p_{ij}(\nu) H_i^*(t_1', \nu) H_j(t_2', \nu) \mathrm{d}\nu, \tag{2}$$

where $p_{ij}(\nu)$ is a non-negative, Fourier-transformable function, and $H_i^*(t_1', \nu)H_j(t_2', \nu)$ is an arbitrary kernel, and the symbol $*$ denotes its complex conjugate.

As a generalization of the scalar model in [24], let us express $H_i(t', \nu)$ and $p_{ij}(\nu)$ as

$$H_i(t', \nu) = A_i \exp\left(-\frac{t'^2}{2T_0^2}\right) \exp(-2\pi i \nu t'), \tag{3}$$

$$p_{ij}(\nu) = B_{ij} T_{cij} \,\mathrm{rect}(T_{cij}\nu), \quad (i = x, y; \; j = x, y), \tag{4}$$

where A_i is the amplitude of the field component, $B_{ij} = |B_{ij}|e^{i\varphi_{ij}}$ is the correlation coefficient, T_0 is the pulse duration, and T_{cij} measures the temporal correlation width between E_i and E_j determining the temporal degree of coherence of the pulse. The function $\mathrm{rect}(x)$ is the rectangular function, which equals 1 for $|x| \le \frac{1}{2}$ and 0 otherwise.

The mutual coherence matrix of a physically realizable field must be quasi-Hermitian, i.e. that $\Gamma_{ij}(t_1', t_2') = \Gamma_{ji}^*(t_1', t_2')$. It is sufficient that this condition is valid if

$$B_{xx} = B_{yy} = 1, \qquad |B_{xy}| = |B_{yx}|, \qquad T_{cxy} = T_{cyx}. \tag{5}$$

Furthermore, the function $p_{ij}(\nu)$ must be non-negative definite [4], and the following inequalities have to be satisfied for any ν:

$$p_{ii}(\nu) \ge 0, \tag{6}$$

$$p_{xx}(\nu)p_{yy}(\nu) - p_{xy}(\nu)p_{yx}(\nu) \ge 0. \tag{7}$$

From Eq. (4), we can readily find that the inequality (Eq. (6)) always holds. By substituting it into inequality (7), we have

$$T_{cxx}T_{cyy}\,\mathrm{rect}(T_{cxx}\nu)\,\mathrm{rect}(T_{cyy}\nu) \ge |B_{xy}|^2 T_{cxy}^2 \left[\mathrm{rect}(T_{cxy}\nu)\right]^2. \tag{8}$$

Since function $\mathrm{rect}(ax)$ equals to 1 for $|x| \le \frac{1}{2a}$ and 0 otherwise, Eq. (8) can be rewritten to obtain the following condition given by

$$\max\{T_{cxx}, T_{cyy}\} \le T_{cxy} \le \frac{\sqrt{T_{cxx}T_{cyy}}}{|B_{xy}|}. \tag{9}$$

By substituting Eqs. (3) and (4) into Eq. (2), we obtain the elements of the mutual coherence matrix of an electromagnetic pulse based on the sinc Gaussian-Schell model correlation distribution, which gives

$$\Gamma_{ij}^{(0)}(t_1', t_2') = A_i A_i B_{ij} \exp\left(-\frac{t_1'^2 + t_2'^2}{2T_0^2}\right) \gamma_{ij}^{(0)}(t_1' - t_2'), \tag{10}$$

$$\gamma_{ij}^{(0)}(t_1' - t_2') = \mathrm{sinc}\left(\frac{t_1' - t_2'}{T_{cij}}\right), \tag{11}$$

and $\mathrm{sinc}(x) = \sin(\pi x)/\pi x$ is the sinc function. Here, $\gamma_{xy}^{(0)}(t_1' - t_2')$ denotes the temporal degree of correlation between the components E_x at time t_1' and E_y at time t_2' as shown in Fig. 1(a). Fig. 1(b) plots the Gaussian intensity distribution of the incident pulse.

We will now investigate the propagation properties of EM SSM pulses in the second-order dispersive media. Suppose an EM SSM pulse source generates a field propagation into half-space $z > 0$ close to the positive z direction in dispersive media. The elements

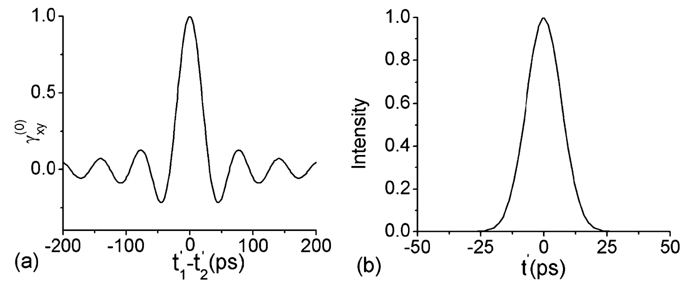

Fig. 1. (a) Temporal degree of correlation between the components E_x and E_y of an EM SSM pulse given by Eq. (5) with $T_{cxy} = 10$ ps. (b) Normalized average intensity distribution $I^{(0)}(t')$ of the pulse with $T_{cxx} = 10$ ps and $T_{cyy} = 25$ ps.

of the mutual coherence matrix can be characterized by the generalized Collins formula in the temporal domain [14]:

$$\begin{aligned}\Gamma_{ij}(t_1, t_2, z) &= \frac{1}{2\pi\beta_2 z} \int\int \Gamma_{ij}^{(0)}(t_1', t_2') \exp\left\{\frac{i}{2\beta_2 z}\left[\left(t_2'^2 - t_1'^2\right) + \left(t_1^2 - t_2^2\right)\right.\right.\\ &\quad \left.\left. - 2\left(t_1' t_1 - t_2' t_2\right)\right]\right\} \mathrm{d}t_1' \mathrm{d}t_2',\end{aligned} \tag{12}$$

with β_2 representing the group velocity dispersion coefficient. In our formulation, we have assumed that the time coordinate is measured in the reference frame moving at the group velocity of the pulse.

Substituting Eq. (2) into Eq. (12), and interchanging the orders of the integrals, we obtain

$$\Gamma_{ij}(t_1, t_2, z) = \int p_{ij}(\nu) H_i^*(t_1, z, \nu) H_j(t_2, z, \nu) \mathrm{d}\nu, \tag{13}$$

where

$$\begin{aligned}H_i^*(t_1, z, \nu) H_j(t_2, z, \nu) &= \frac{1}{2\pi\beta_2 z} \int\int H_i^*(t_1', \nu) H_j(t_2', \nu) \exp\left\{\frac{i}{2\beta_2 z}\left[\left(t_2'^2 - t_1'^2\right)\right.\right.\\ &\quad \left.\left. + \left(t_1^2 - t_2^2\right) - 2\left(t_1' t_1 - t_2' t_2\right)\right]\right\} \mathrm{d}t_1' \mathrm{d}t_2'.\end{aligned} \tag{14}$$

By substituting Eq. (3) into Eq. (14), a lengthy but straightforward derivation will lead to

$$\begin{aligned}H_i^*(t_1, z, \nu) H_j(t_2, z, \nu) &= \frac{A_i A_j T_0}{\Delta(z)} \exp\left[-\frac{T_0^2}{4\beta_2^2 z^2}(t_1 - t_2)^2 + \frac{i}{2\beta_2 z}\left(t_1^2 - t_2^2\right)\right]\\ &\quad \times \exp\left\{-\left[\frac{t_1 + t_2}{2} + \frac{iT_0^2}{2\beta_2 z}(t_1 - t_2) - 2\pi\beta_2 z\nu\right]^2 / \Delta^2(z)\right\},\end{aligned} \tag{15}$$

where

$$\Delta^2(z) = T_0^2 + \frac{\beta_2^2 z^2}{T_0^2}. \tag{16}$$

The elements of the matrix (13) can be found by solving Eq. (15) together with Eq. (4) using numerical integration. The intensity and the degree of polarization of the EM SSM pulse can be calculated by the expressions [1,2]

$$I(t, z) = \mathrm{Tr}\, \overleftrightarrow{\Gamma}(t, t, z), \tag{17}$$

$$P(t, z) = \sqrt{1 - \frac{4\,\mathrm{Det}[\overleftrightarrow{\Gamma}(t, t, z)]}{\mathrm{Tr}[\overleftrightarrow{\Gamma}(t, t, z)]^2}}. \tag{18}$$

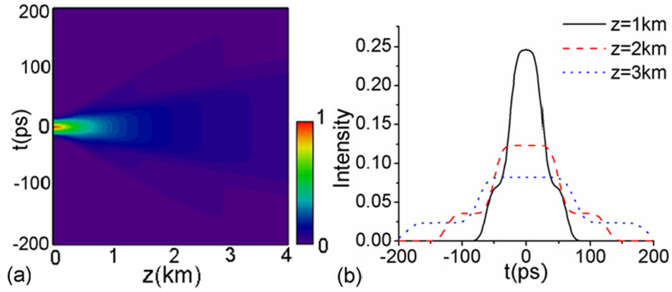

Fig. 2. (a) Normalized average intensity distribution $I(t,z)$ of an EM SSM pulse with $T_{cxx}=10$ ps and $T_{cyy}=25$ ps as function of propagation distance z and time t. (b) Pulse shapes at selected propagation distances.

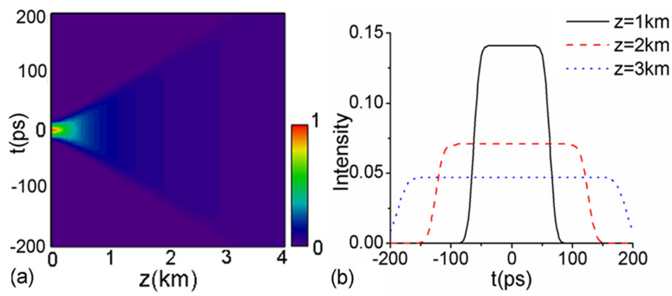

Fig. 3. As Fig. 2 but for $T_{cxx}=T_{cyy}=10$ ps.

where Det and Tr stand for determinant and trace of the matrix, respectively.

3. Numerical calculations

We will now analyze numerically the temporal evolution of an EM SSM pulse in dispersive media. Unless specified in the captions, the parameters for the following calculations are chosen as $A_x = A_y = 1$, $|B_{xy}| = 0.2$, $T_0 = 10$ ps, $\beta_2 = 50$ ps^2/km.

Fig. 2 illustrates the temporal evolution of the intensity distribution of the EM SSM pulse with $T_{cxx}=10$ ps and $T_{cyy}=25$ ps in 2D and 3D versions. Compared to the source intensity distribution presented in Fig. 1(b), it can be seen that the Gaussian profile of the source pulse gradually transforms into a double-layer flat-top distribution with increasing propagation distance. Due to the fact that the coherence properties of the pulse source plane are intimately related to the intensity distribution at the far field, hence for the pulse with different temporal correlation widths ($T_{cxx} \neq T_{cyy}$), the far-field temporal intensity distributions of x and y directions are different, which results in the appearance of double-layer profile. However, for the case of $T_{cxx}=T_{cyy}$ as shown in Fig. 3, the same correlation widths of field components results in two directions with the same temporal intensity distributions, and posses a single-layer intensity profile.

Fig. 4(a) exhibits the behavior of the temporal intensity distributions at a fixed propagation distance $z=1$ km for different temporal correlation widths T_{cyy}. One can clearly see that as the difference of correlations between field components increases, the double-layer profile becomes increasingly apparent as expected. Fig. 4(b) shows that the dispersion coefficient of the medium may markedly affect the intensity profile, both on the height and the width.

In the following, we will investigate the influence of the correlation widths on the temporal degree of polarization at a fixed propagation distance. As can be seen from Fig. 5(a), the EM SSM pulses with the most general initial correlation $T_{cxx} \neq T_{cyy}$, are capable of forming an inverted flat temporal polarization distribution with adjustable height and edge sharpness, which essentially

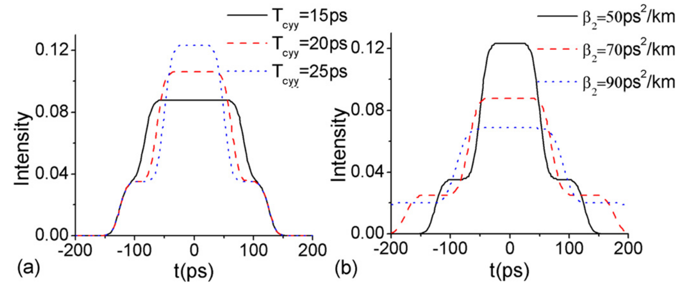

Fig. 4. Normalized average intensity distribution $I(t,z)$ of an EM SSM pulse with $T_{cxx}=10$ ps in dispersive media at the distance $z=1$ km as a function of t: (a) for different T_{cyy} with $\beta_2=50$ ps^2/km, (b) for different β_2 with $T_{cyy}=25$ ps.

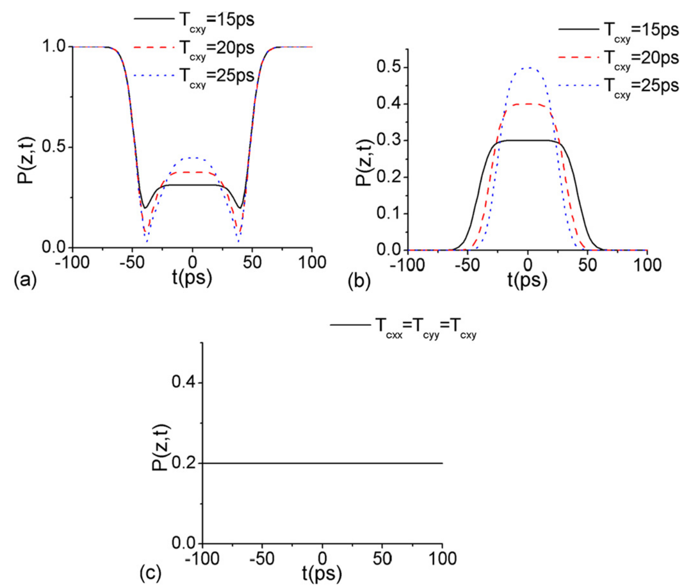

Fig. 5. Temporal degree of polarization of an EM SSM pulse with different correlation parameters in dispersive media at the distance $z=1$ km as a function of t. (a) for different T_{cxy} with $T_{cxx}=10$ ps and $T_{cyy}=15$ ps, (b) for different T_{cxy} with $T_{cxx}=T_{cyy}=10$ ps, (c) $T_{cxx}=T_{cyy}=T_{cxy}=10$ ps.

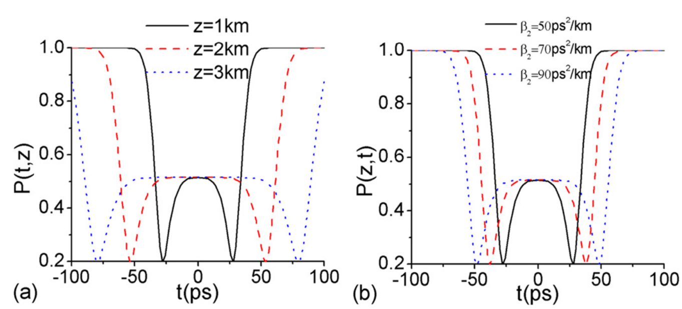

Fig. 6. Temporal degree of polarization of an EM SSM pulse with $T_{cxx}=10$ ps and $T_{cyy}=T_{cxy}=25$ ps in dispersive media. (a) for different z with $\beta_2=50$ ps^2/km, (b) for different β_2 with $z=1$ km.

is attributed to the unique coherence properties of the source. For the case $T_{cxx}=T_{cyy}$ shown as Fig. 5(b), the temporal distribution of the degree of polarization presents a flat profile, and it may gradually convert into a Gaussian profile with the increase of T_{cxy}. Note that for the special case $T_{cxx}=T_{cyy}=T_{cxy}$ shown in Fig. 5(c), the degree of polarization does not change upon propagation, and it remains the same as the source at any time, such a trend is in agreement with results previously reported (see, for example, [11]).

From Fig. 6(a) one finds that, the inverted flat polarization profile is preserved for any propagation distance in the medium, which becomes wider with the increasing value of z. In Fig. 6(b), we show the effect of the dispersion coefficient on the temporal polarization profile. One can see that the flat prominent profiles at different values of β_2 have the same height and also the steepness of edges, but different widths. Moreover, as β_2 increases, the width of the flat profile becomes larger.

4. Conclusion

In summary, we have introduced a new class of electromagnetic random pulses with sinc Schell-model temporal correlation distribution. We have derived the critical conditions for such pulses to be realized in practical setting. We have also studied the temporal average intensity and the temporal degree of polarization of the EM SSM pulse in dispersive media based on the weighted superposition method. The numerical results have demonstrated that, the EM SSM pulse can produce a double-layer flat-top temporal intensity profile except the particular case of $T_{cxx} = T_{cyy}$, and the height and diameter of two layer distribution can be controlled by adjusting the temporal coherent widths of the source. It is also shown that the control of the temporal polarization profile at certain propagation distance may be fulfilled by manipulating the coherent properties of the source. Finally, the temporal evolution of the EM SSM pulse alters significantly with the change in the dispersion coefficient. Such novel pulses carry great potential application in pulse shaping and pulsed laser material processing.

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