Space charge limited current in a gap combined of free space and solid

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The paper presents a model of the space charge limited (SCL) electron conduction in a gap with a combination of free space and dielectric solid. The SCL electron current density J is solved numerically to obtain the voltage scaling of $J \sim V_G^n$, and n is found to be between $n=3/2$ (a vacuum gap) and $n=2$ (a dielectric diode). The dependence of n is calculated as a function of dielectric constant, electron mobility, and the relative length scale between the free space and dielectric solid. The model has been used to explain a recent experiment. *© 2010 American Institute of Physics*.
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Charge injection into a solid dielectric can be classified into low and high charge injection. In the low charge injection regime, the amount of injection is considerably lower than the intrinsic concentration of the solid, and the current-voltage (*IV*) characteristic obeys the Ohm's law. At high charge injection regime, the *IV* characteristic is influenced by the self-electric field created by the injection, and it is known as the space charge limited (SCL) conduction. For a one dimensional (1D) trap-free solid, SCL electron current density is described by the classical Mott–Gurney (MG) law, as follows:[1]

$$J_{\mathrm{MG}} = \frac{9}{8}\mu_n \epsilon_0 \epsilon_r \frac{V_G^2}{L^3}, \qquad (1)$$

where μ_n is the electron mobility in solid, ϵ_0 is free space permittivity, ϵ_r is relative (dielectric) permittivity, V_G is the applied voltage, and L is the length of the solid.

There are renewal interests in SCL conduction found in many devices, such as such as GaN nanorod,[2] organic device,[3] polymer transistor,[4] nanowire,[5] magnetoresistance,[6] and silicon Schottky junction.[7] Analytical two-dimensional MG law was also developed recently for Ohmic contact[8] and Schottky contact.[9]

For a vacuum (or free space) gap, the equivalent SCL electron current density is given by the Child–Langmuir (CL) law, as follows:[10,11]

$$J_{\mathrm{CL}} = \frac{4\epsilon_0}{9}\sqrt{\frac{2q}{m_e}}\frac{V_G^{3/2}}{L^2}, \qquad (2)$$

where q and m_e are the charge and mass of the free electron, respectively. The study of CL law is also an area of active research in the development of non-neutral plasma physics, high current diodes, high power microwave sources, vacuum microelectronics, and sheath physics. Recent developments include analytical multidimensional models, contact properties, quantum tunneling, and ultrafast time scale.[12–15]

While the transition between the MG and CL law can be formulated by controlling the effect of collisions in the gap,[8] there is no model yet to address the SCL electron transport in a gap which is consisted of both solid and free space. In this case, the voltage scaling (n) of $J \sim V_G^n$ is neither $n=2$ nor $n=3/2$ as predicted by the MG law and CL law, respectively. For example, a recent experiment has reported $n=1.65$ for SCL conduction measured by STM with a finite vacuum region between the STM tip and the sample.[16]

Thus it is of interests to develop a model to determine the scaling of n in a such a combined gap with both free space and solid, and the value of n will be dependent on various parameters such as relative dielectric constant, mobility, gap length, and the relative length scale between the solid and free space regions. In our model, we will present two cases, respectively, showing the SCL electron injections into the free space region first before entering the dielectric solid, and vice versa.

For the first case, we consider a gap consisted of a free space region and a dielectric solid, which is, respectively, located from $x=0$ to $x=x_1$, and from $x=x_1$ to $x=L$. The electrons are injected from the grounded cathode at $x=0$ to the anode at $x=L$ with an applied voltage of V_G. In the free space region ($x=0$ to $x=x_1$), we use the standard derivation of CL law in solving the Poisson equation, current continuity equation and energy balance condition, which gives the SCL current density in the form of the 1D CL law, as follows:

$$J = \frac{4\epsilon_0}{9}\sqrt{\frac{2q}{m_e}}\frac{\psi_{x_1}^{3/2}}{x_1^2}. \qquad (3)$$

Here ψ_{x_1} is the electrostatic potential at the interface ($x=x_1$), which is related to the electric field at the interface by,

$$E(x_1) = \sqrt{\frac{2J}{\epsilon_0}\sqrt{\frac{2m_e}{q}}\psi_{x_1}}. \qquad (4)$$

In the dielectric region ($x=x_1$ to $x=L$), we combined the Poisson equation and the drift current equation $[J=qn(x)\times\mu_n\times E(x)]$ to obtain $dE(x)/dx=J/[\mu_n E(x)\epsilon_r\epsilon_0]$, which has a solution of $E^2(x)-E^2(x_1)=2J(x-x_1)/[\mu_n\epsilon_r\epsilon_0]$. The last equation can be solved by integrating $E(x)=d\psi_x/dx$ from $x=x_1$ to $x=L$ with the respective boundary conditions of $\psi_x=\psi_{x_1}$ and $\psi_x=V_G$. In doing so, we obtain another relation between $E(x_1)$ and ψ_{x_1} given by,

$$V_G - \psi_{x_1} = \frac{\mu_n\epsilon_r\epsilon_0}{3J}\left\{\left[\frac{2J}{\mu_n\epsilon_r\epsilon_0}(L-x_1)+E^2(x_1)\right]^{3/2} - E^3(x_1)\right\}. \qquad (5)$$

Thus for a given L, x_1, μ_n, and ϵ_r, we solve Eqs. (3)–(5) numerically to obtain the SCL current density J as a function of V_G. For simplicity, the value of dielectric constant is fixed

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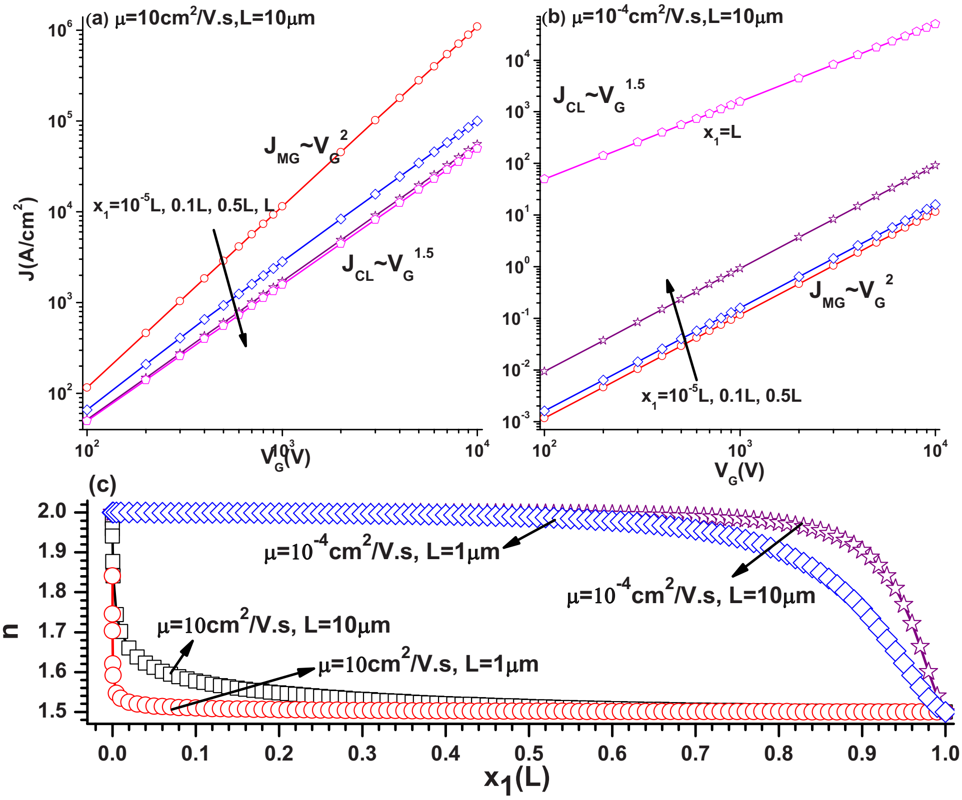

FIG. 1. (Color online) Dependence of J as a function of V_G for SCL electron injection into a gap of free space ($x=0$ to $x=x_1$) and dielectric ($x=x_1$ to $x=L$) for various $x_1/L=10^{-5}$, 0.1, 0.5, 1 at $L=10$ μm and $\mu_n=$ (a) 10 and (b) 10^{-4} cm^2/V s. The arrow indicates the values of x_1 in an increasing order. (c) The scaling of n for $J\sim V_G^n$ as a function of x_1 for different L and μ_n.

at $\epsilon_r=11.7$, and the value of μ_n is varied in our calculation, unless mentioned otherwise in the paper. While we have chosen $\epsilon_r=11.7$ with $\mu_n=10^{-4}$ to 10 cm^2/V s to present our model, the findings (see below) remain valid for any fixed value of $\epsilon_r \times \mu_n$.

In Figs. 1(a) and 1(b), the resulting SCL current density J as a function of V_G for x_1/L ($=10^{-5}$, 0.1, 0.5, and 1) at $L=10$ μm, is shown, respectively, for μ_n ($=10$ and 10^{-4} cm^2/V s). The scaling of $J\sim V_G^n$ between $n=2$ and 3/2 can be obtained by numerical fitting at arbitrary value of x_1. For example, we obtain the MG law ($J\sim V_G^2$) with $n=2$ at small $x_1/L=10^{-5}$ as the gap becomes a solid diode. At $x_1=L$, we recover the CL law ($J\sim V_G^{3/2}$) with $n=3/2$ as it is a vacuum gap. The magnitude of J is however dependent on the mobility used in the model. For high mobility case ($\mu_n=10$ cm^2/V s), we have a higher MG law as compared to the CL law as shown in Fig. 1(a). On the other hand, CL law is higher than the MG law for low mobility case $\mu_n=10^{-4}$ cm^2/V s [see Fig. 1(b)].

In order to clearly show the transition between the MG law and CL law, the n scaling of $J\sim V_G^n$ is shown in Fig. 1(c) as a function of x_1 for $L=1$ and 10 μm, and $\mu_n=10$ and 10^{-4} cm^2/V s. When the mobility is high (see red circle and black square symbols), the value of the MG law is much higher than the CL law, and we have sharp transition from $n=2$ to $n=1.5$ at small $x_1/L\approx 0$. If the mobility value is low (see blue diamond and purple star symbols), MG law is much smaller than the CL law, and the transition is from $n=2$ to $n=1.5$ at large $x_1/L>0.8$. The effect of gap length L is moderate as compared to the one due to the mobility.

For the second case to be shown below, we reverse the location of the free space and dielectric, such that the electrons are injected into the dielectric region ($x=0$ to $x=x_1$) first before entering the free space region ($x=x_1$ to $x=L$). In this case, the SCL current density in the dielectric region (according to the MG law) is,

$$J=\frac{9}{8}\mu_n\epsilon_0\epsilon_r\frac{\psi_{x_1}^2}{x_1^3}. \qquad (6)$$

The electric potential at the interface ($x=x_1$) is ψ_{x_1} with an related electric field of,

$$E(x_1)=\sqrt{\frac{2J}{\mu_n\epsilon_r\epsilon_0}x_1}. \qquad (7)$$

In the free space charge region ($x=x_1$ to $x=L$), we follow the standard derivation of the CL law, to obtain the electric field in the form of the following:

$$\frac{d\psi(x)}{dx}=\left[\frac{2J}{\epsilon_0}\sqrt{\frac{2m_e}{q}(\psi(x)-\psi_{x_1})}+E^2(x_1)\right]^{1/2}. \qquad (8)$$

Equations (6)–(8) can be solved numerically with the boundary conditions of $\psi(x)=\psi_{x1}$ at $x=x_1$ and $\psi(x)=V_G$ at $x=L$. In doing so, the SCL current density J as a function of V_G can be obtained numerically for a given L, x_1, μ_n, and ϵ_r.

For the second case, the results of J as a function of V_G for various x_1 are shown in Figs. 2(a) and 2(b) with the same parameters used in Fig. 1. Here, we have $n=3/2$ (CL law) at small $x_1/L=10^{-5}$, and $n=2$ (MG law) at large $x_1/L=1$. If the mobility is high ($\mu_n=10$ cm^2/V s), MG law is higher than the CL law as shown in Fig. 2(a). For low mobility case ($\mu_n=10^{-4}$ cm^2/V s), we have higher CL law [see Fig. 2(b)]. The scaling of n in $J\sim V_G^n$ dependence is also shown as a function of x_1 in Fig. 2(c). From the figure, the transition is from $n=3/2$ at small $x_1/L\approx 0$ (nearly a vacuum gap) to $n=3/2$ at large $x_1/L=1$ (a dielectric diode). At low mobility case, it behaves like a dielectric diode with $n=2$ except in the range of very small $x_1/L\approx 0$. At high mobility case, the tran-

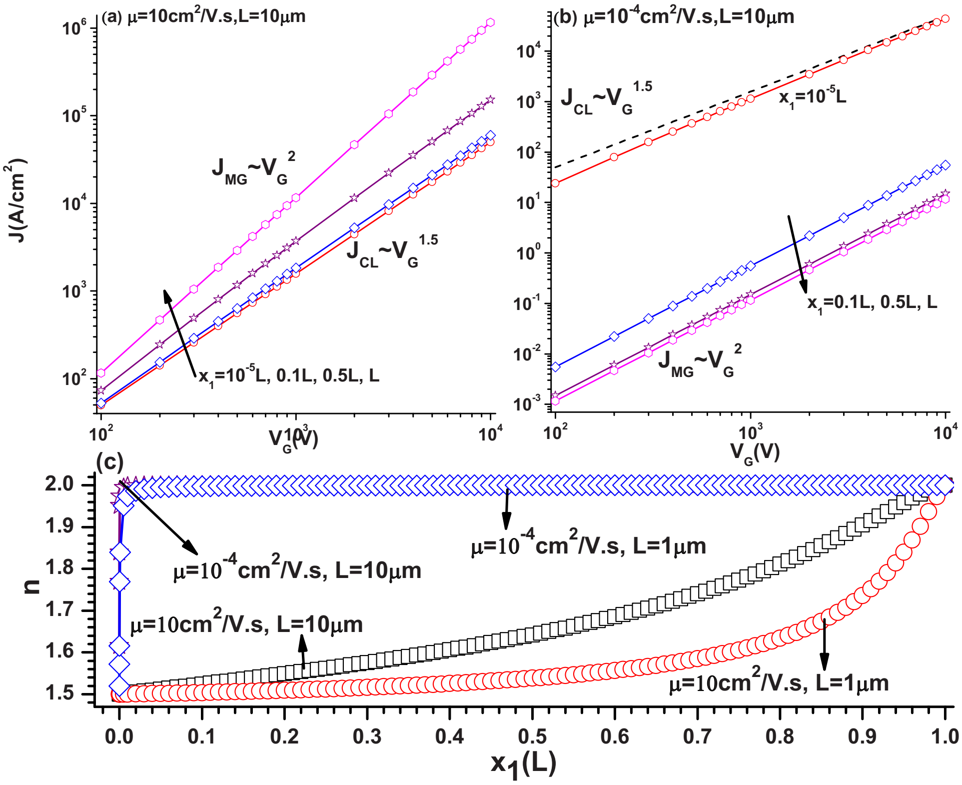

FIG. 2. (Color online) Dependence of J as a function of V_G for SCL electron injection into a gap of solid ($x=0$ to $x=x_1$) and free space ($x=x_1$ to $x=L$) for various $x_1/L=10^{-5}$, 0.1, 0.5, 1 at $L=10$ μm and $\mu_n=$ (a) 10 and (b) 10^{-4} cm^2/V s. The arrow indicates the values of x_1 in an increasing order. (c) The scaling of n for $J\sim V_G^n$ as a function of x_1 for different L and μ_n.

sition from $n=3/2$ to 2 is moderate as a function of x_1. The effect of the gap length L is more significant at high mobility case as compared to the low mobility case.

In a recent paper,[16] an experiment was performed to understand the physical mechanism of the nitrogen incorporation in the high-κ dielectric hafnium oxide (HfO_2). It was reported that SCL conduction was measured by using the scanning tunneling microscopy (STM). The experimental measurement showed a current-voltage (I-V) of $I\sim V^n$ with $n=1.65$ in the range of $I=0.4$ nA at $V=4$ V to $I=1.8$ nA at $V=10$ V (see Fig. 7 in Ref. 16). Note the SCL electron current is injected into the dielectric layers and then collected by the STM tip with a circular area of 10 nm in radius.

The deviation of the reported $n=1.65$ from $n=2$ (MG law) is due to the finite vacuum region of 0.5 nm between the STM tip and the dielectric layers consisted of HfO_2 (5.1 nm) and SiO_x (1.6 nm) after the nitrogen annealing. Using the experimental parameters of $x_1=5.1+1.6=6.7$ nm, and $L=x_1+0.5$ nm$=7.2$ nm, we are able to reproduce $n=1.65$ by assuming $\epsilon_r\times\mu_n\approx 2$ to 3×10^{-4} (with μ_n in the unit of square centimeter per volt second). This corresponds to $\mu_n\approx10^{-5}$ cm^2/V s at $\epsilon_r=25$ (dielectric constant of HfO_2). For comparison, the dependence of the calculated n as a function of $\epsilon_r\times\mu_n$ is shown in Fig. 3.

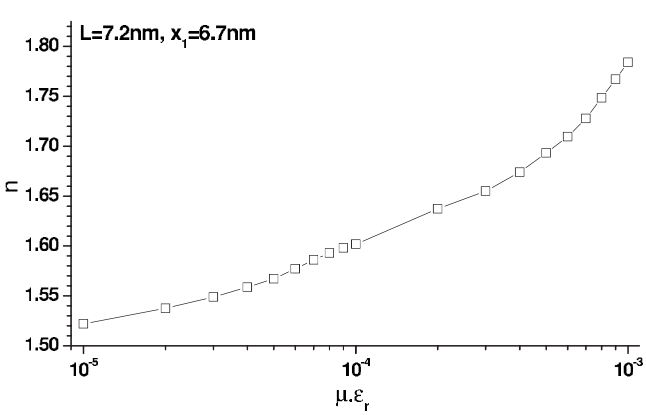

FIG. 3. The dependence of n as a function of $\epsilon_r\times\mu_n$ (in square centimeter per volt second) in linear-log scale for the case of the STM tip experiment with $L=7.2$ nm and $x_1=6.7$ nm reported in a recent paper (Ref. 16).

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