Two-dimensional analytical Mott-Gurney law for a trap-filled solid

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The letter presents a two-dimensional analytical model of the space charge limited (SCL) current injection in a solid with exponentially distributed trap energy state. By considering that the electrons are injected from an infinitely long emission strip of width W, the one-dimensional SCL current density is enhanced by a factor of $1+F(4/\pi)/(W/L)$, where $F=1/(l+2)$ measures the mean position of the injected electrons in the solid of length L, and l is the ratio of the distribution of the traps to the free carriers. The analytical formula is verified by using a two-dimensional device simulator. © *2007 American Institute of Physics*. [DOI: 10.1063/1.2721382]

For high current transport in a medium, where the space charge effects of the injected current are important, the current is generally termed as space charge limited (SCL) current. In the one-dimensional (1D) planar gap (free space), the SCL current density is given by the classical Child-Langmuir (CL) law:[1]

$$J_{\mathrm{CL}}=\frac{4\epsilon_0}{9}\sqrt{\frac{2e}{m_e}}\frac{V^{3/2}}{L^2}, \tag{1}$$

where L is the gap spacing, V is the gap voltage, ϵ_0 is the free space permittivity, and e and m_e are the charge and mass of the electron, respectively. The corresponding SCL current in a trap-free solid is known as the Mott-Gurney law:[2]

$$J_{\mathrm{MG}}=\frac{9}{8}\mu\epsilon_r\epsilon_0\frac{V^2}{L^3}, \tag{2}$$

where ϵ_r is the relative permittivity, and μ is the electron mobility.

In comparison to the recent developments of CL law in multidimensional classical models,[3–5] quantum models,[6,7] and short-pulse models,[8,9] the 1D MG law is relatively unexplored in multidimensional models. Various improvements had been made in the 1D MG law, which includes influence of trapped charges, temperature dependence, transient behavior, and transition to 1D CL model.[10–14] Recently, there are renewed interests in the studies and applications of MG law in various materials, such as SCL current fluctuations in organic semiconductors,[15] SCL photocurrent in polymers and fullerenes,[16] polymer SCL transistor,[17] and SCL current in nanowires.[18] In such studies, the length of the solid can be comparable or larger than its cross section that the 1D MG law may not be valid. Thus, in this study, we will develop a two-dimensional (2D) analytical MG law in a solid with exponentially distributed trap energy state.

Similar to the 2D CL law,[3–5] we assume that the enhancement of the 1D MG law for uniform charge injection into a solid is expressed as (in terms of the 1D model)

$$\frac{J_{\mathrm{MG}}[2\mathrm{D}]}{J_{\mathrm{MG}}[1\mathrm{D}]}=1+FG. \tag{3}$$

The parameter $F\equiv\int_0^L(x/L)n(x)dx/\int_0^L n(x)dx$ measures the normalized mean position of the injected electrons in the solid of length L, and $n(x)$ is the electron density in the solid, which is calculated by the 1D MG model (with and without traps). Here, G is a correction parameter, which depends on the geometrical properties of the emission area (between the cathode and the trap-filled solid). For simplicity, we will consider that the electrons are injected from an infinitely long strip of finite width W, which corresponds to $G=(4/\pi)/(W/L)$.[5]

For a trap-filled solid between the cathode and anode, we consider that the energy state of the traps is described by an exponential function, $N_t(E)=(N_t/kT_c)\exp[(E-E_c)/kT_c]$, characterized by a constant T_c, where $N_t(E)$ is the density of trap states as a function of band energy E, E_c is the energy at the bottom of the conduction band, k is the Boltzmann constant, and N_t is the total number of trap density integrated from the quasi-Fermi energy level to the intrinsic Fermi energy level. By assuming that the total trapped electron density N_t is much larger than the free electron density, the trap-limited SCL current density J_{TL} for a solid with exponentially distributed traps is[2]

$$J_{\mathrm{TL}}=N_c\mu e^{1-l}\left[\frac{\epsilon_r\epsilon_0 l}{N_t(l+1)}\right]^l\left(\frac{2l+1}{l+1}\right)^{(l+1)}\frac{V^{(l+1)}}{L^{(2l+1)}}, \tag{4}$$

where N_c is the effective density of states at $E=E_c$, and $l=T_c/T(\geqslant 1)$ is the ratio of distribution of traps to the free carriers. With $n(x)\propto x^{-l/(l+1)}$, we have $F=1/(l+2)$; the enhancement of the 2D MG law for a trap-filled solid will depend on the value of l, which characterizes the energy distribution of traps. For $l=1$, we recover the shallow trap limit of $F=1/3$, which is also identical for a trap-free solid.

It is also of interest to study the smooth transition of F between the 2D MG law and 2D CL law. For simplicity, we consider a trap-free solid and solve the force equation, Poisson equation, and current continuity equation in the normalized form of[14]

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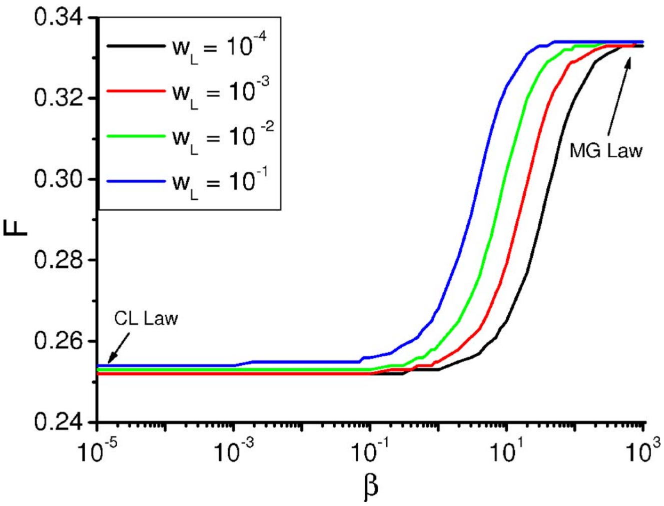

FIG. 1. (Color online) Dependence of the normalized mean position F of the electron density as a function of β for various w_L=0.1 to 10^{-4} (left to right) for a trap-free solid.

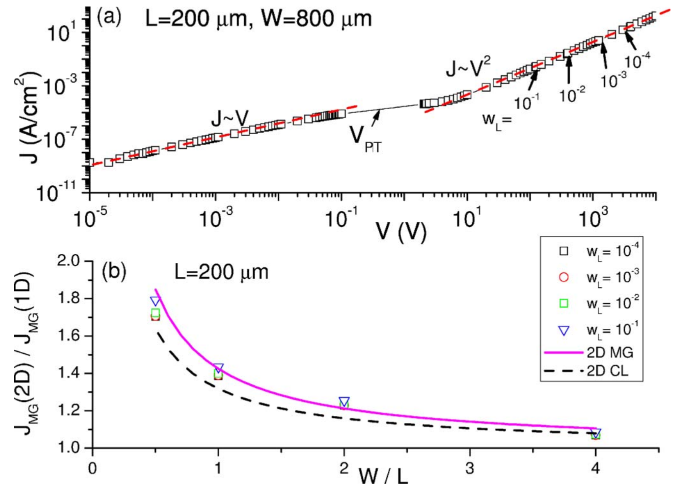

FIG. 2. (Color online) (a) SCL current density J obtained from the MEDICI simulator as a function of applied voltage V at fixed W/L=4 for L=200 μm (without traps). The red dashed line shows the transition from the Ohmic region ($J \propto V$) to the trap-free MG law $J \propto V^2$ around the punch-through voltage $V_{\rm PT}$. The corresponding V at various $w_L=10^{-4}$, 10^{-3}, 10^{-2}, and 10^{-1} used for comparison with the analytical 2D scaling [in (b)] are indicated. (b) Comparison of simulation results (symbols) with the 2D trap-free MG law (solid) and the 2D CL law (dashed) as a function of W/L=0.5–4 for L=200 μm.

$$\frac{d^2u}{dw^2} + \frac{\beta}{\sqrt{u}}\frac{du}{dw} + 1 - \frac{1}{\sqrt{u}} = 0. \tag{5}$$

Here, u is the dimensionless squared velocity$=(en_0 v)^2/j^2$, $w=x/x_0$ is the dimensionless distance, $x_0=j/(\sqrt{2}en_0\omega_p)$ is the characteristic length, $\omega_p=\sqrt{(e^2n_0)/(\epsilon_0\epsilon_r m^*)}$ is the plasma frequency, $\beta=(\sqrt{2}\omega_p\tau_m)^{-1}$ is the dimensionless collision frequency, m^* is the effective electron mass, v is the electron velocity, τ_m is the momentum relaxation time, n_0 is the intrinsic or doping density of the solid, and e is the electronic charge. By using the boundary conditions of u=0 and du/dw=0 at w=0 (cathode) for space charge current injection, the equation is solved numerically for $n(x)$ in order to calculate $F=\int_0^{w_L} wu^{-1/2}dw / w_L \int_0^{w_L} u^{-1/2}dw$, at given values of β and $w_L=L/x_o(<1)$. The condition of $w_L<1$ is imposed to ensure high current injection for which the amount of injected charges is higher than the intrinsic charge. At the anode ($w=w_L$), the relation of the applied voltage V with other parameters is $V=2V_{\rm PT}[u_L+2\beta\int_0^{w_L} u^{1/2}dw]/w_L^2$, where $V_{\rm PT}=(en_oL^2)/(2\epsilon_0\epsilon_r)$ is the punch-through voltage that the space charge has extended through the solid, and u_L is the solution of $u(w)$ at $w=w_L$ obtained from solving Eq. (5). Thus the applied voltage has to be larger than $V_{\rm PT}$ in order to observe the trap-free MG law, else the electron transport will obey Ohm's law (see Fig. 2).

Figure 1 shows the calculated F as a function of β at different w_L=0.1 to 10^{-4} (left to right). It is clear that there is a smooth transition from F=1/3 at large β value (trap-free MG law or collision-dominated SCL current) to F=1/4 at small β (CL law or ballistic SCL current). The transition region lies within the range of β=0.1–100, and the exact value of F will depend on β and w_L.

To verify the analytical 2D trap-free MG law with F=1/3, we used a 2D device simulator called MEDICI (Ref. 19) to create a structure of N^+-i-N^+ of silicon (Si) of width W and length L with heavily doped cathode and anode in order to have ohmic contact. The intrinsic region between them has an intrinsic uniform electron density of about 1.5×10^{16} cm^{-3} and a uniform fixed mobility (μ) determined at β=1000 through the relationship of $\mu=e\tau_m/m^*$. The other parameters used in the simulation are m^*/m_e=0.31 and ϵ_r=11.8.

The current density-voltage (J-V) characteristic obtained from MEDICI is plotted in Fig. 2(a) at a fixed W/L=4 for L=200 μm, which shows the transition from Ohm's law ($J\sim V$) to MG law ($J\sim V^2$) around the punch-through voltage $V_{\rm PT}$ of about 0.4 V. For various values of $w_L=10^{-4}$–0.1, we determined the corresponding applied voltage to each w_L as indicated in the figure. To determine the 2D MG law as a function of W/L, MEDICI is used to obtain the current density by varying W at fixed L=200 μm. The comparison between the simulation results (symbols) and the analytical 2D trap-free MG law of F=1/3 (solid lines) is plotted in Fig. 2(b) as a function of W/L=0.5–4. The analytical results agree very well with the simulation; even at very small W/L=0.01, the error is less than 5%, (not shown). For comparison, the analytical 2D CL law (ballistic transport) of F=1/4 is also plotted (dashed lines). Note that the MEDICI simulator cannot be used to verify the transition region ($1/4<F<1/3$) between the MG law and the CL law, as diffusive transport has been assumed in the simulator.

To verify the enhancement factor of $F=1/(l+2)$ for the 2D trap-limited MG law, the same simulator is used at L=10 μm for two kinds of trap distribution: l=2 and l=3, with N_t=(1 or 5)$\times10^{17}$ cm^{-3}. The distribution of traps used in the simulation are created by discretizing the band energy based on the exponential function. Due to the limitation of the simulator, only 50 energy step sizes can be used.

In Fig. 3(a), we show the transition from Ohm's law to a trap-limited region at $V_{\Omega\text{-trap}}$, and from the trap-limited region to the trap-free-like MG law at $V_{\text{trap-MG}}$ for the case of l=2 and $N_t=10^{17}$ cm^{-3}. The trap-limited region is confined between the voltage range of $V_{\Omega\text{-trap}}\approx0.85$ V to $V_{\text{trap-MG}}$ $\approx$29 V, as indicated in the figure. A similar transition had also been observed in other cases, but at different voltage ranges. For the l=2 and $N_t=5\times10^{17}$ cm^{-3} case, the range is

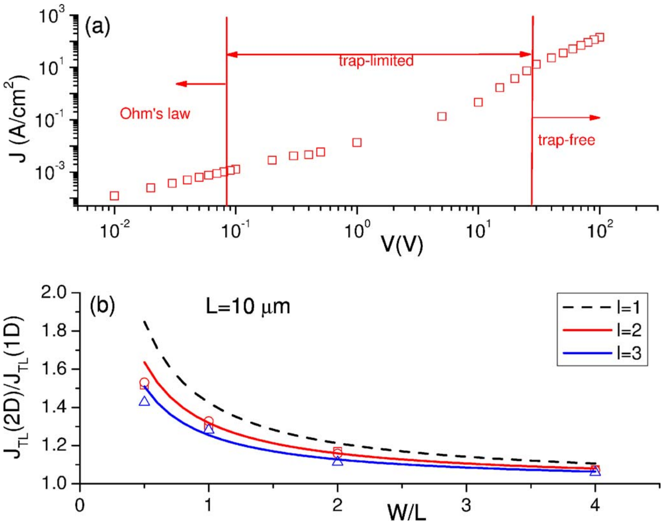

FIG. 3. (Color online) (a) SCL current density J obtained from the MEDICI simulator as a function of applied voltage V at $W/L=4$ and $L=10$ μm for a trap-filled solid of exponentially distributed traps for $l=2$ and $N_t=10^{17}$ cm^{-3} (red square). (b) Normalized 2D trap-limited current density as a function of $W/L=0.5-4$ at $L=10$ μm for three simulated cases in their respective trap-limited regions: $l=2$ with $N_t=10^{17}$ cm^{-3} at 10 V (red square), $l=2$ with $N_t=5\times10^{17}$ cm^{-3} at 200 V (red circle), and $l=3$ with $N_t=10^{17}$ cm^{-3} at 300 V (blue triangle). The lines are the analytical solutions at $l=2$ and 3 (solid) and $l=1$ (dashed).

from $V_{\Omega\text{-trap}}\approx 1.25$ V to $V_{\text{trap-MG}}\approx 750$ V. For the $l=3$ and $N_t=10^{17}$ cm^{-3} case, we have $V_{\Omega\text{-trap}}\approx 8$ V and $V_{\text{trap-MG}}\approx 480$ V. Note that the corresponding crossover voltage has been estimated by using $V_{\Omega\text{-trap}}=(en_0L^2/\epsilon_0\epsilon_r)(l+1/2l+1)^{[(l+1)/l]}(l+1/l)\Gamma$ and $V_{\text{trap-MG}}=(en_0L^2/\epsilon_0\epsilon_r)\Gamma^l[(9/8)((l+1)/l)^l(l+1/2l+1)^{l+1}]^{[1/(l-1)]}$, where $\Gamma=(N_t/n_0)(n_0/N_c)^{1/l}$.[2,20]

In comparison with the analytical formula of $F=1/(l+2)$, we plot the normalized 2D trap-limited MG law in terms of its 1D limit [see Eq. (4)] as a function of $W/L=0.5-4$ in Fig. 3(b). The comparison shows good agreement between the simulation results (symbols) and the anaytical equations (solid lines) at $l=2$ and 3. Note that the simulation results are obtained from a particular voltage in the trap-limited region. This comparison shows that the simple 2D scaling laws derived in this study remain valid even for a trap-limited solid. The enhancement is directly related to the trap distribution characterized by the value of l, and thus confirms the analytical scaling of $F=1/(l+2)$. For completeness, the formula of shallow trap limit at $l=1$ is also plotted in dashed line.

In conclusion, we have presented a simple analytical 2D MG law for SCL electron flows in a trap-filled solid (with a simple 2D shape), which is confirmed by a 2D device simulator. The effects of traps are included by assuming an exponentially distributed trap energy state. Smooth transition from the 2D MG law and the 2D CL law are also demonstrated at the trap-free limit. This result may be useful in determining the amount of maximum SCL current density in novel 2D structures such as nanotubes, organic semiconductors, and nanowires that have been reported to be operating at SCL regime.

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