Wavelet Analysis of the Turbulent LES Data of the Lid-Driven Cavity Flow

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Abstract. Both Fourier and wavelet transforms are performed on data obtained from large-eddy simulations of the turbulent flow in a lid-driven cubical cavity. The analyzed data or synthetic signals are picked at three specific points inside the cavity allowing to investigate three regimes over time: laminar, transitional and turbulent. The main objective of this study is to generate and analyze synthetic signals in order to confirm the correlation between the computed physical quantities and those expected theoretically.

1 Introduction

The analysis of sampled signals obtained from experiments and direct numerical simulations (DNS) of turbulent fluid flows through wavelet analysis is now common

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practice. Such analysis often provides tremendous insight into the flow behavior otherwise difficult, if not impossible, to apprehend with more conventional statistical signal analysis methods (e.g. Fourier transforms). The use of wavelet analysis to study signals from large-eddy simulations (LES) is not as common, mainly because of the intrinsic high level of non-physical noise introduced by the subgrid models. In addition and in comparison with experiments and DNS, LES not only have a reduced resolution in space but also in time. However, depending on the subgrid model and the numerical method used, these difficulties may be overcome. In these conditions, a time-frequency analysis, and in particular, its subdiscipline wavelet analysis could provide a 'local' analysis of transient turbulent events. Such an approach has been employed and is reported here for the study of the time histories of the pressure and other fluctuating quantities in the locally-turbulent lid-driven cubical cavity flow.

2 LES of the Lid-Driven Cubical Cavity Flow

Large-eddy simulations of the turbulent flow in a lid-driven cubical cavity have been carried out at a Reynolds number of 12 000 using the Legendre spectral element method [2]. Two distinct subgrid-scales models, namely a dynamic Smagorinsky model and a dynamic mixed model, have been both implemented and used to perform long-lasting simulations required by the relevant time scales of the flow. The resolution in time is high enough so that subgrid modeling is only needed to account for the reduced resolution in space. Practically, it is usually more cost-effective to be under-resolved in space than in time. In addition, being under-resolved both in space and time leads to unavoidable numerical instabilities, e.g. CFL criterion not respected, etc. All filtering levels make use of explicit filters applied in the physical space (on an element-by-element approach) and spectral (modal) spaces. The two subgrid-scales models had been validated and compared to available experimental and numerical reference results, showing very good agreement. Specific features of lid-driven cavity flows in the turbulent regime, such as inhomogeneity of turbulence, turbulence production near the downstream corner eddy, small-scales localization and helical properties have been investigated and discussed in the large-eddy simulation framework [2]. Time histories of quantities such as the total energy, total turbulent kinetic energy or helicity exhibit different evolutions but only after a relatively long transient period. At a Reynolds number of 12 000, the lid-driven cavity flow is in the locally-turbulent regime and is proved to be highly inhomogeneous in the secondary-corner regions of the cavity where turbulence production and dissipation are important [2]. The maximum production of turbulence was found to be located in the downstream-corner-eddy region just above the bottom wall.

3 Spectral Analysis and Wavelet Analysis

The time series of three quantities, namely pressure, local kinetic energy and x-component of the velocity field, have been extracted from the LES databases at three distinct locations inside the cavity. Each extraction point has been purposedly chosen to characterize the three different regimes—laminar, transitional, turbulent—encountered within the lid-driven cavity flow at Re = 12 000. The Cartesian coordinates of these three points, inside the cavity, corresponding to a larminar, transitional and turbulent region of the flow are respectively given by $(0,0,0)$, $(0.65,-0.6,-0.6)$ and $(0.78,-0.94,-0.337)$ in units of the cavity length[2]. The groundbreaking work of Kolmogorov [5, 6] highlight the fact that the velocity fluctuations of a turbulent flow can be analyzed and characterized based upon the behaviors of the scales of the spatial increment of the Eulerian velocity or the temporal increment of the Lagrangian velocity. The central postulate of statistical isotropy of small spatial and temporal scales, used by Kolmogorov in [5] is connected to the independence of these small scales with respect to the mechanism of injection of energy which occurs for the large eddies. In addition, the statistical analysis of the temporal fluctuations of the pressure field allows one to study the vortically-intense regions of the flow [1]. Given the fact that the pressure field is the solution of a Poisson equation (the flow being incompressible), the vortical structures are therefore directly connected to the rapid changes the temporal signal of the pressure field, see Fig. 1.

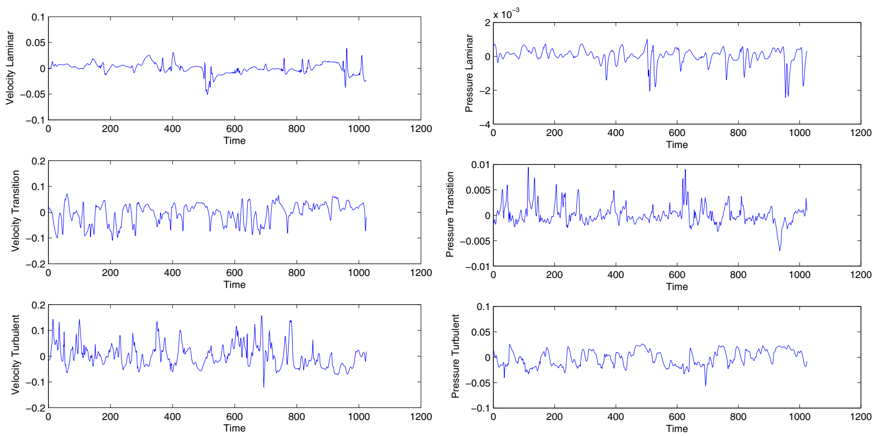

Fig. 1 *x*-component of the velocity field (left column) and the pressure field (right column) for the three regimes

A first relevant analysis to give an overall validation of the simulated system consists in the Fourier transform computation applied to the velocity signal and to the pressure signal (Fig. 1) at the point of maximum turbulence production within the cavity. The Fourier transform can be written as

$$S(\nu) = \int s(t)e^{-2i\pi\nu t}\,\mathrm{d}t, \quad (1)$$

where $s(t)$ is the analyzed signal, t the time and ν the frequency. The results of spectral analysis for the intensity (square modulus of the quantity)in a Log-Log scale are shown in Fig. 2 where a $-5/3$ (resp. $-7/3$) slope is observed for the velocity (resp. pressure) signal. These results which are characteristics of the developed turbulence region of the cavity flow are in good agreement with those predicted by Kolmogorov's statistical theory of the turbulence K41 [5, 9]. Slopes were conventionally estimated using a linear regression method.

A second appropriate analysis consists in the continuous wavelet transform analysis [7]. The wavelet transform is expressed by

$$C_{\Psi}[s](a,b) = \langle \Psi_{a,b}, s \rangle = \int \overline{\Psi}_{a,b}(t) s(t)\,\mathrm{d}t, \quad \text{with} \quad \Psi_{a,b} = \frac{1}{\sqrt{a}} \Psi\left(\frac{t-b}{a}\right), \qquad (2)$$

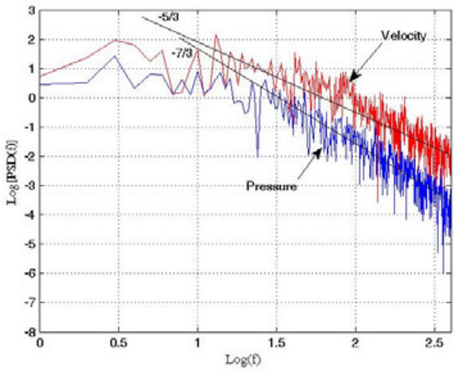

Fig. 2 Fourier transform of the time histories of the velocity and pressure signals at the point of maximum turbulence production within the cavity

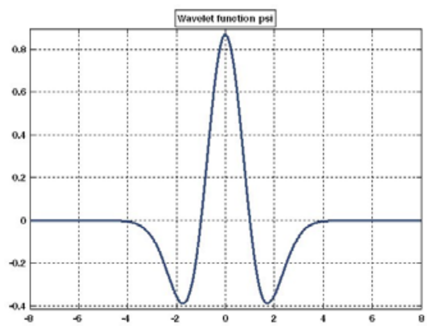

Fig. 3 Mexican hat wavelet

and corresponds to the inner product of the signal $s(t)$ with the successive versions of the mother wavelet $\Psi_{a,b}$, where a is a real positive parameter, b a real parameter, and the overline denotes the complex conjugate. The selected wavelet must verify the following admissibility condition $\int \Psi(t)\,\mathrm{d}t = 0$. In the sequel, all wavelet analyses are based on the so-called 'Mexican hat' wavelet (Fig. 3) expressed by

$$\Psi(t) = \frac{2\pi^{-1/4}}{\sqrt{3}}(1-t^2)e^{-t^2/2}. \qquad (3)$$

This wavelet is directly connected to the negative normalized second derivative of a Gaussian (Fig. 3). This wavelet is C^∞ and is well localized in the time domain as well as in the frequency domain. In addition, its two first moments vanish and verify

$$\int t^q \Psi(t)\,\mathrm{d}t = 0, \qquad 0 \le q < N. \tag{4}$$

This property is essential in analyzing the singularities within a signal [1].

The continuous wavelet transform $C_\Psi[s](a,b)$ is now a conventional tool for the analysis of singularities in a signal at an instant t_0, and hence allows to expand the concept of 'singularity exponent'. It is well-known that the wavelet transform, near a singular point $b = t_0$, behaves like a power law according to the scale with the Hölder exponent $h(t_0)$

$$|C_\Psi[s](a,t_0)| \propto a^{h(t_0)}, \tag{5}$$

where the Hölder exponent $h(t_0)$ is a measure of the strength of the singularity [1]. In the present framework, given the (inhomogeneous) production of turbulence within the cavity flow [2], a *multi-resolution analysis* of some signals extracted from the LES database is well suited. The results of the wavelet transform computation are traditionally presented in a graph with a horizontal axis for the time, a vertical one for the scale a and flooded color contours of the continuous wavelet transform. As a first step, the continuous wavelet transform of the LES signals is used to reveal the changing patterns between the three regimes, laminar, transitional and turbulent. The wavelet analysis of the pressure signal appears in Fig. 4 and in Fig. 5. Figure 4 given in grey levels, reveals the difference of the signal amplitudes considering the three regimes. Figure 5 highlights the gradual emergence of 'time-scale' patterns at large scales. In addition, these patterns grow in size when moving from the transitional regime to the turbulent one as shown in Fig. 6.

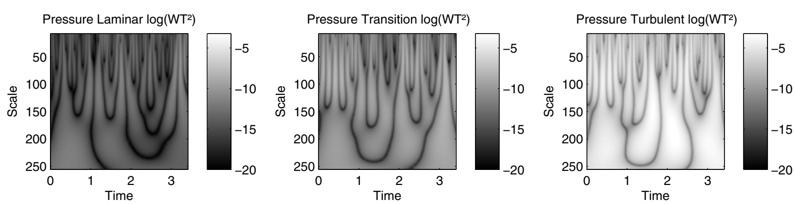

Fig. 4 Logarithm of the square of the continuous wavelet transform of the pressure field at the three locations of the cavity corresponding to laminar, transitional and turbulent regimes, from left to right respectively

To quantify the turbulence, a powerful yet straightforward technique consists in implementing the wavelet transform modulus maxima method [1]. More specifically, one aims at obtaining the skeleton defined by lines of maxima of the wavelet transform modulus, which is calculated for the pressure signal in both laminar and turbulent regimes. The results are shown in Fig. 7 where filaments appear in both

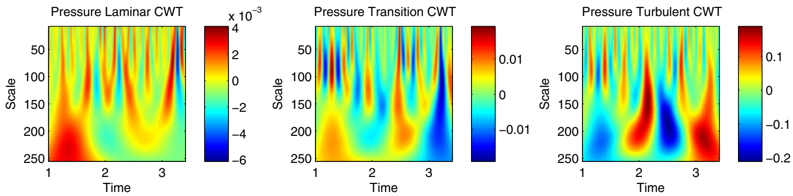

Fig. 5 Continuous wavelet transform of the pressure field at the three locations of the cavity corresponding to laminar, transitional and turbulent regimes, from left to right respectively. Red (resp. blue) represents positive (resp. negative) contour levels

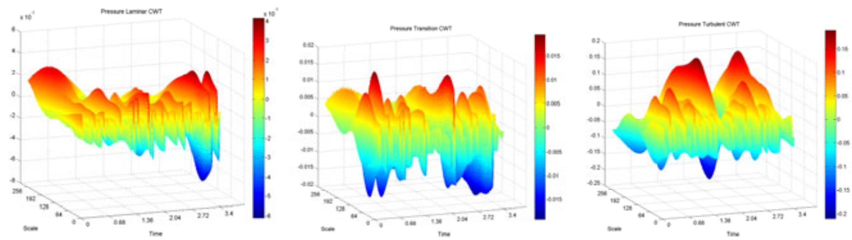

Fig. 6 3D wavelet transform of the pressure for the laminar (Left), the transitional (Center) and the turbulent (Right) regimes

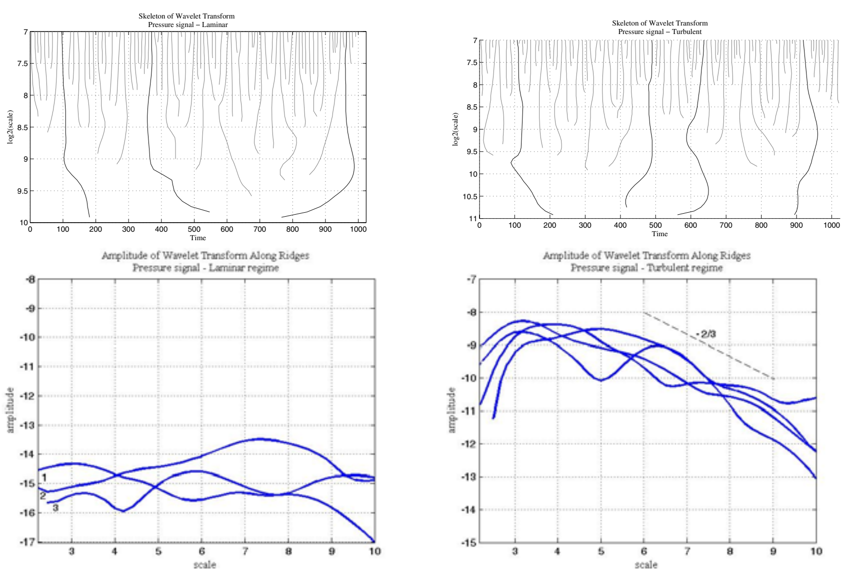

Fig. 7 Top row: skeleton of the wavelet transform of the pressure field for both the laminar (left column) and turbulent (right column) regimes. Bottom row: amplitude of the wavelet transform along the longest filaments of the skeleton

cases. The longest filaments are collected and represented in Fig. 7 (bottom row) where the scale is on the horizontal axis and the magnitude of the filaments on the vertical one. The laminar filaments appear almost constant in amplitude, whereas for the ones for the turbulent regime highlight a $-2/3$ slope in the large scales. This $-2/3$ slope corresponds to very large depressions associated with the turbulence burst occurring when a pair of counter-rotating vortices is produced. This pair of counter-rotating vortices has been identified as the highly-coherent vortical structure responsible for the production of turbulence [2]. Herein, it is identified by very strong singularities characterized by a negative Hölder exponent.

The above results for both the transitional and turbulent regions of the flow provide structures possessing a clear fractal signature, which is also related to the high level of singularities in the Hölder spectra.

4 Towards a Spectrum of Singularities

To take into account all singularities in the LES signal, one has to resort to a *multi-fractal model* and hence study the spectrum of Hölder exponent, also known as 'spectrum of singularities' [9]. A common technique consists in applying the wavelet transform modulus maxima method (WTMM). Unlike the 'traditional' technique employed in Sec. 3, the WTMM allows one to estimate the spectrum of singularities without accounting for the singularities of negative Hölder exponent [1].

Considering a multi-fractal formalism [9], the K41 theory [5] leads to a statistical homogeneous velocity field characterized by only one Hölder exponent when the Reynolds number tends to infinity. For small scales, the properties of scale invariance of the Navier-Stokes equations are statistically preserved. But when considering intermittency phenomena in fluid flow, the hypothesis of homogeneity falls because of these small scales . An improvement of the earlier statistical theory of homogeneous and isotropic turbulence K41 [5] is given by the KO62 [6, 8] theory which allows to consider intermittent phenomena. Similarity assumptions (statistical relations between local fluctuations of the velocity field and fluctuations of the dissipative energy field) in the case of isotropic turbulence can be understood within the multi-fractal framework [1, 3, 4, 7]. However, the computation of the spectrum of singularities requires signals of several hundred thousand samples, which are unfortunately not available at the present time. On-going computations should be able to feed the system in order to achieve higher statistical sampling and hence calculate the spectrum of singularities $D(h)$ by a multi-fractal formalism based on the wavelet transform modulus maxima method [1].

5 Conclusions

In the present work, wavelet analysis and other methods derived from it are employed to treat LES signals and to characterize the locally turbulent flow in a lid-driven cubical cavity. Despite the inherently low space and time resolution of the LES signals, the wavelet analysis reveals some very interesting features through its Hölder exponent in a multi-fractal framework. Comparisons of these results for three different locations corresponding to three different turbulent regimes show the effectiveness of this approach for a locally inhomogeneous and anisotropic turbulent flow such as the one in the lid-driven cavity at a Reynolds number of 12 000. This first step allows to consider a more general and systematic investigation and characterization of physical quantities computed through LES in other flow configurations. Comparisons with a similar analysis of the same flow but obtained by DNS [10] is now considered.

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