Peak-to-Average Ratio Constrained Demand-Side Management with Consumer’s Preference in Residential Smart Grid

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Abstract—In a smart grid network, demand-side management plays a significant role in allowing consumers, incentivized by utilities, to manage their energy consumption. This can be done through shifting consumption to off-peak hours and thus reducing the peak-to-average ratio (PAR) of the electricity system. In this paper, we begin by proposing a demand-side energy consumption scheduling scheme for household appliances that considers a PAR constraint. An initial optimization problem is formulated to minimize the energy cost of the consumers through the determination of the optimal usage power and operation time of throttleable and shiftable appliances, respectively. We realize that the acceptance of consumers of these load management schemes is crucial to its success. Hence, we then introduce a multi-objective optimization problem which not only minimizes the energy cost but also minimizes the inconvenience posed to consumers. In addition to solving the proposed optimization problems in a centralized manner, two distributed algorithms for the initial and the multi-objective optimization problems are also proposed. Simulation results show that the proposed demand-side energy consumption schedule can provide an effective approach to reducing total energy costs while simultaneously considering PAR constraints and consumers’ preferences.

Index Terms—Demand-side management, peak-to-average ratio, energy consumption scheduling scheme, consumer’s preference, smart grid.

I. INTRODUCTION

Generally, traditional power grids are used to distribute power from a few central generators to a large number of consumers. Although adequate to meet past challenges, renewable integration, distributed generation, demand management, have increasingly exposed the limitations of the traditional grid systems. In contrast, the Smart Grid (SG) system uses two-way flows of electricity and information to create an automated and distributed advanced energy delivery network [1]-[4]. By utilizing adaptive signal processing, statistical signal processing and modern information technologies, [5]-[8], the SG is able to distribute power in more efficient ways and respond to wide ranging conditions and variational changes. Broadly stated, the residential SG could respond to events that occur anywhere in the grid that could impact generation, transmission, distribution, and consumption, and then adopt the corresponding strategies to cope with them [9]-[13].

Made possible by advanced communication technologies, a key advantage of the SG is flexible bi-directional demand side management (DSM) for residential smart grid [14]-[17]. Benefiting from information exchange between supply and demand, DSM is able to reduce peak electricity loads and increase the reliability of the power grid. Utilities and system operators can effectively manage power generation and through incentives, encourage the shifting of high-energy demand household appliances to off-peak hours [18]. Cost sensitive consumers are able to adjust their demand according to time differentiated electricity pricing and make appropriate consumption scheduling decisions [19]-[22]. Recently, there have been several studies detailing DSM approaches in residential grid networks [23]-[29].

In [23], an incentive-based energy consumption scheduling scheme was proposed to schedule the power of individual appliances to reduce the energy cost and peak-to-average ratio (PAR). Both centralized optimization based on linear programming and decentralized game theoretic approaches have also been proposed. PAR provides a measure on how peak electricity consumption affects the system, particularly in efficiency and reliability. A reduction in peak generally improves the electricity generation efficiency of the system and this effect can be seen in [24]. If we further assume that the overall energy consumption of the system remains constant, a reduction in PAR would also result in the reduction of the magnitude of peak energy demand. This reduction can increase system reliability through increasing spare capacity in the event of supply tightness, which was one of the main contributing factors in the Californian electricity crisis.
of 2000 [25]. Inspired by [23], [26] proposed a demand-side management mechanism by considering the fixed power consumption pattern of household appliances. The proposed mechanism optimized the energy consumption schedule to satisfy the specific requirements of the individual household appliances. [27] presented demand-side management strategies for a large number of the household appliances of several types. Besides the load shifting management of the shiftable household appliances, another management strategy is the control of energy usage by controlling throttleable appliances. These appliances are defined as appliances which have a fixed operational period but flexible power consumption pattern [28] [29]. In [30] [31] the authors focused on energy consumption scheduling to reduce the energy cost by considering the consumer related consumption preferences. However, the specific dissatisfaction functions for both of shiftable appliances and throttleable appliances are not considered. In addition, how to yield the dissatisfaction functions to fit various cases when the different user has different preference is still an open issue.

The focus of this paper is on demand-side energy consumption scheduling scheme for both shiftable and throttleable appliances in terms of constrained PAR. Under this constraint, a uniform load demand during the day can be guaranteed. In addition, the consumers’ own preferred usage requirements are addressed in our energy scheduling algorithm. When considering the reschedule of shiftable appliances to reduce electricity costs, consumers may have a limited window in which they will accept an operational shift. We define this delay, operation delay, as a metric for measuring the amount of operational delay a consumer is willing to endure. Similarly, for throttleable appliances, the power gap is used to measure the consumers’ willingness to endure reduced consumption; here, it is defined as the difference between the consumer’s preferred power and the scheduled power. Considering both of the metrics in the optimization problem should allow for better consideration of the consumers’ usage preferences.

Hence, in this paper, we propose an improved energy consumption* scheduling scheme for household appliances under the supply-demand framework of the residential smart grid. Three types of appliances are considered: shiftable, throttleable and essential appliances, where essential appliances correspond to those appliances that are non-shiftable and non-throttleable. The initial objective of our proposed scheduling scheme is to seek the optimal response strategy for each type of appliances in order to minimize the total costs associated with electricity consumption under the PAR constraint. An initial optimization problem of minimizing the energy cost is formulated in order to find the optimal energy consumption and operation time of the throttleable and shiftable appliances, respectively. Subsequently, a multi-objective optimization problem considering consumer preferences is formulated, with objectives to minimize the energy cost, operation delay and energy gap. In addition to solving the optimal problems in a centralized manner at residential scheduler (RS) level, two distributed algorithms for the initial optimization problem and the multi-objective optimization problem are also proposed. The distributed algorithms can find the near-optimal schedules with minimal information exchange between the RS and consumers. Simulation results show that the proposed scheduling scheme can achieve effective scheduling under a PAR constraint considering different levels of consumer preferences.

The rest of the paper is organized as follows. In Section II, the system model is introduced. The initial energy cost minimization problem for the residential smart grid system is proposed and solved in both centralized and distributed formulations in Section III. Section IV studies the energy cost minimization problem with consumer’s preference and provides a distributed subgradient algorithm to solve this problem. Section V presents the simulation results of the proposed algorithms. Finally, we conclude the paper in Section VI.

II. SYSTEM MODEL

A. Residential Grid Networks

We consider the residential smart grid in which the energy from the power grid is shared by several consumers (homes) through the power line, as shown in Fig.1. Each consumer is equipped with a smart meter, which not only distributes the electricity from the power line to all appliances in each home, but also collects each consumer’s demand and preference information. Through the communication line, the smart meters are capable of reporting the collected information to the RS. In a centralized manner, the RS can globally optimize the energy consumption and schedule all household appliances based on the collected information. In a distributed manner, the RS can provide the variation information of the entire grid networks to each consumer for local scheduling process. Here, we conduct a theoretical study by assuming that information of a day is known before hand. We will later in the future work design algorithm based on only historical or statistical information, and the result of this paper can be serve as a performance bound for a practical scenario.

The appliances in the residential grid network are categorized into three types: shiftable, throttleable and essential appliances. The scheduling process would only affect the shiftable and throttleable appliances. For shiftable appliances, such as dishwashers, washing machines and clothes dryers, the operational period can be delayed while power consumption for these appliances would not be adjusted during operation. For throttleable appliances with fixed operational periods such as refrigerators and air conditioning units, their power consumption can be adjusted within a range during their fixed operational period. For essential appliances such as TVs, electric stoves and lamps, which have a fixed power requirement and operational period, a continuous supply of power should be ensured throughout the optimization period.

B. Load Demand Description

Let $\mathcal{N}$ denote the set of consumers, where the number of consumers is $\mathcal{N} \triangleq |\mathcal{N}|$. For each consumer $n \in \mathcal{N}$, let $\mathcal{M}_n = \{T_n \cup \mathcal{S}_n \cup \mathcal{R}_n\}$ denote the set of household
appliances, where $\mathcal{I}_n$, $\mathcal{S}_n$ and $\mathcal{R}_n$ denote the set of essential, shiftable and throttleable appliances, respectively. Assume that the number of household appliances in each consumer is $M \triangleq |\mathcal{M}_n|$. Then, we define the energy consumption scheduling vector for these three types of the appliances as

$$
e_{n,i} \triangleq [e_{n,i}^1, \ldots, e_{n,i}^t, \ldots, e_{n,i}^T],$$

$$
e_{n,s} \triangleq [e_{n,s}^1, \ldots, e_{n,s}^t, \ldots, e_{n,s}^T],$$

$$
e_{n,r} \triangleq [e_{n,r}^1, \ldots, e_{n,r}^t, \ldots, e_{n,r}^T],$$

where scalar $e_{n,i}^t$, $e_{n,s}^t$ and $e_{n,r}^t$ denote the corresponding energy consumption that is scheduled for essential appliances $i \in \mathcal{I}_n$, shiftable appliance $s \in \mathcal{S}_n$ and throttleable appliance $r \in \mathcal{R}_n$ by consumer $n$ at unit time interval $t$, respectively.

Let $L_{n,t}$ denote the total load of consumer $n$ at each time interval $t \in T \triangleq \{1, \ldots, T\}$, where $T$ is the total number of all unit time intervals. The load for consumer $n$ is denoted by $\{L_{n,t}\}_{t=1}^T \triangleq \{L_{n,1}, \ldots, L_{n,T}\}$. Moreover, the total load of the $n$th consumer at $t$ is obtained as

$$L_{n,t} = \sum_{i,s,r \in \mathcal{A}_n} e_{n,i}^t + e_{n,s}^t + e_{n,r}^t, t \in T. $$

Based on these definitions, the total load across all consumers at time interval $t \in T$ can be calculated as

$$L_t \triangleq \sum_{n \in \mathcal{N}} L_{n,t}. $$

Let $L_p$ and $L_a$ denote the peak load and average load of the residential smart grid, respectively. We have

$$L_p = \max_{t \in T} L_t$$

and

$$L_a = \frac{1}{T} \sum_{t \in T} L_t. $$

Hence, the PAR of the load demand, denoted by $\Gamma_{PAR}$, can be obtained as [23]

$$\Gamma_{PAR} = \frac{L_p}{L_a} = \frac{T \max_{t \in T} L_t}{\sum_{t \in T} L_t}. $$

### C. Energy Cost Function

In this subsection, we try to abstract a general energy cost function from some practical electricity pricing schemes to reflect the cost of electricity consumption to the consumer. Without loss of generality, we consider three electricity price models [32] [33], which represent three tariffs commonly used by utilities: flat rate model (FRM), two-step piecewise model (TPM) and five-tier pricing model (FPM). The first model assumes a constant price for consumers regardless of their energy consumption. The latter two models employ increasing tiered pricing schemes for which the base level has the lowest price and the price for the consumers increases in one (TPM) and four (FPM) additional tiers.

Let $\mathcal{R}(L_t)$ denote the energy cost for the consumers under the pricing scheme at each time interval $t$. Three energy cost functions which can represent the cost tariffs are given by:

**FRM:**

$$\mathcal{R}(L_t) = a_t^{FRM} L_t, \quad (1)$$

where $a_t^{FRM}$ is a constant.

**TPM:** The energy cost under this model can be approximated by the quadratic function given in [26]

$$\mathcal{R}(L_t) = a_t^{TPM} L_t^2 + b_t^{TPM} L_t + c_t^{TPM}, \quad (2)$$

where $a_t^{TPM} > 0$ and $b_t^{TPM}, c_t^{TPM} \geq 0$ at each time interval $t$.

**FPM:** The energy cost in this model can also be approximated by the quadratic function with different coefficients.

$$\mathcal{R}(L_t) = a_t^{FPM} L_t^2 + b_t^{FPM} L_t + c_t^{FPM}, \quad (3)$$

where $a_t^{FPM} > 0$ and $b_t^{FPM}, c_t^{FPM} \geq 0$ at each time interval $t$.

Fig. 2 shows the actual price model and the approximated cost functions for TPM and FPM. Assume that $a_t^{TPM} = 0.2$ cents, $b_t^{TPM} = c_t^{TPM} = 0$; and $a_t^{FPM} = 0.7$ cents, $b_t^{FPM} = c_t^{FPM} = 0$. We observe that the quadratic function has appropriate approximations for the practical price models. Hence, in this paper, we employ the quadratic function as the cost function for residential energy consumption. Meanwhile, we consider a time varying electricity tariff model through varying the value of $a_t$ for different time intervals.

### III. INITIAL ENERGY COST MINIMIZATION PROBLEM

#### A. Energy Cost Minimization

In this subsection, our goal is to schedule the operational period and power consumption of the appliances in order to achieve minimum energy costs. Since the RS has no impact on the schedule of energy consumption for essential appliances, the essential load $E_t = \sum_{i \in \mathcal{E}_t} \sum_{n \in \mathcal{N}} \sum_{t \in \mathcal{I}_n} e_{n,i}^t$ has constant value in the objective function of our problem.

For shiftable appliances, we mainly focus on designing the energy consumption scheduler to systematically shift the operational period of the appliance. In this case, the consumer needs to pre-set the start $\alpha_{n,s} \in \mathcal{T}$ and end $\beta_{n,s} \in \mathcal{T}$ of a time period that the appliance can be scheduled. For each consumer $n$ and each shiftable appliance $s$, we denote the total energy consumption as $E_{n,s}$. Then, we have
conditioning unit at the preferred power level is the consumed power by the air-
and may not accept a thermostat level lower than its typical usage pattern. For example, consumers may refers to the consumed power of the appliance used according
barely satisfies the consumer’s requirement. Preference power
power consumed by each appliance operating at a level that
n
and maximum power for each appliance
\[ p_{n,s}^{max} \]
are the energy consumption constraints of the shiftable and the
throttleable appliances of the
\[ T \]
consumers less than
\[ \Gamma \]
means perfectly flat load profile which might

\[ e_t^{n,s} = 0, \quad \forall t \in T \setminus T_{n,s}, \quad (5) \]
where \( p_{n,s}^{max} \) is the maximum power level for s shiftable appliance in consumer n and \( T_{n,s} = [\alpha_{n,s}, ..., \beta_{n,s}] \). For each shiftable appliance, the time period assigned by the RS needs to be larger than or equal to the time interval needed to finish the operation.

For throttleable appliances, the energy consumption scheduler does not aim to change the appliance’s operational period, but instead adjusts the operational power demand of each throttleable appliance within a predetermined range at each operation time interval \( t \). We define the tolerance power level \( p_{n,r}^{tol} \) and the preferred power level \( p_{n,r}^{pre} \) as the minimum and maximum power for each appliance \( r \in R \) for each consumer \( n \in N \). Tolerance power refers to the electrical power consumed by each appliance operating at a level that barely satisfies the consumer’s requirement. Preference power refers to the consumed power of the appliance used according to its typical usage pattern. For example, consumers may prefer a 24°C to a tolerable 26°C of the air-conditioning unit and may not accept a thermostat level lower than 24°C or higher than 26°C. In such a case, the consumed power of the air-conditioner at 26°C is used as the tolerance power level and the preferred power level is the consumed power by the air-conditioning unit at 24°C. Hence we can define the following

\[ p_{n,r}^{tol} \leq e_t^{n,r} \leq p_{n,r}^{pre}, \quad \forall t \in T_{n,s}. \quad (7) \]

We denote the total energy consumption of all throttleable appliances in nth customer as \( E_{n,r} \). We can obtain

\[ \beta_{n,r} \sum_{t = \alpha_{n,r}} e_t^{n,r} \leq E_{n,r}, \quad (8) \]

where \( \alpha_{n,r}, \beta_{n,r} \in T \) and \( \beta_{n,r} - \alpha_{n,r} \in T \) are the beginning and the end of the time interval that throttleable appliances can be operated, \( T_{n,r} = [\alpha_{n,r}, ..., \beta_{n,r}] \). For each throttleable appliance, the time interval is fixed and the appliance should finish its operation during its own time interval.

Therefore, the energy consumption scheduling problem can be formulated in terms of minimizing the energy costs to all consumers and appliances, which can be expressed as the following optimization problem:

\[ \min_{\mathbf{e}} \sum_{t=1}^{T} \mathbb{R}(L_t) \]

\[ \begin{align*}
&= \sum_{t=1}^{T} a_i \left( \sum_{n \in N} \sum_{s \in \delta_n} e_t^{n,i} + \sum_{s \in \delta_n} \sum_{r \in R_n} e_t^{n,s} + \sum_{r \in R_n} e_t^{n,r} \right) \quad (9) \\
\text{s.t.} \quad &\Gamma_{PAR} = \frac{\max_{L_i \in \mathcal{T}} L_i}{\sum_{t=1}^{T} L_t} \leq \Gamma_{max} \\
&\sum_{t = \alpha_{n,s}} e_t^{n,s} = E_{n,s}, \quad e_t^{n,s} = 0, \quad \forall t \in T \setminus T_{n,s}, \\
&\sum_{t = \alpha_{n,r}} e_t^{n,r} \leq E_{n,r}, \quad e_t^{n,r} = 0, \quad \forall t \in T \setminus T_{n,r}, \\
&0 < e_t^{n,s} \leq p_{n,s}^{max}, \quad \forall t \in T_{n,s}, \\
&0 < e_t^{n,r} \leq p_{n,r}^{max}, \quad \forall t \in T_{n,r}. 
\end{align*} \]

The explanation of the constraints in \((P1)\) are as follows: the first constraint restricts the PAR of the load demand from all residences less than \( \Gamma_{max} \). In this paper, we choose \( \Gamma_{max} \) carefully to guarantee the feasibility of solving \((P1)\). It might not be possible to achieve every value of PAR constraint; e.g., \( \Gamma_{max} = 1 \) means perfectly flat load profile which might never be achieved by any algorithm and by any amount of rescheduling. Minimum energy cost obtained by optimal energy consumption schedule with different values of \( \Gamma_{max} \) are shown in Section V.A. The second equality and the third inequality are the energy consumption constraints of the shiftable and the throttleable appliances of the \( n \)th consumer in each operation interval \( t \), respectively. The last two inequalities are the hourly energy consumption constraints of the shiftable appliance \( s \) and the throttleable appliance \( r \) of consumer \( n \) in each operation interval \( t \), respectively.

\[ e_t^{n,r} = 0, \quad \forall t \in T \setminus T_{n,r}, \quad (9) \]

\[ \begin{align*}

\text{B. The Dual of Energy Minimization Problem}

We firstly introduce vector \( \mathbf{e}_n = \{ e_{n,i}, e_{n,s}, e_{n,r} \} \), which is formed by stacking up the energy consumption scheduling vectors for all appliances \( \{ i, s, r \} \in M_n \) of consumer \( n \). Then, we define a feasible energy consumption scheduling set corresponding to \( n \)th consumer as follows:
\[ E_n = \left\{ \{e_{n,i}, e_{n,s}, e_{n,r}\} \mid \beta_{n,s} \begin{array}{c} e_{n,s} = E_{n,s}, \quad e_{n,r} = 0, \quad \forall t \in T \setminus T_{n,s}, \\
\end{array} \right\} \]
\[ \sum_{t = \alpha_{n,r}} e_{n,r} \leq E_{n,r}, \quad e_{n,r} = 0, \quad \forall t \in T \setminus T_{n,r}, \]
\[ 0 < e_{n,s} \leq p_{n,r}^{\max}, \quad \forall t \in T_{n,s}, \]
\[ p_{n,r}^{\text{tot}} \leq e_{n,r} \leq p_{n,r}^{\text{pre}}, \quad \forall t \in T_{n,r}. \]

In the proposed scheduling method, the RS validly controls consumer’s energy consumption only if \( e_n \in E_n \). Therefore, we can simplify problem (P1) based on the following theorem.

**Theorem 1.** If \( L_t = \sum_{n \in N} (\sum_{i \in \mathcal{I}_t} e_{n,i} + \sum_{s \in S_n} e_{n,s} + \sum_{r \in \mathcal{R}_n} e_{n,r}) \) and \( E = \frac{1}{T} \left( E_i + \sum_{n \in N} \sum_{s \in S_n} E_{n,s} + T \sum_{n \in N} \sum_{r \in \mathcal{R}_n} p_{n,r}^{\text{pre}} \right) \), the problem (P1) can be rewritten as

\[
\min_{e_n} \sum_{t = 1}^{T} \mathcal{R}(L_t) \\
\text{s.t.} \quad L_t \leq \Gamma_{\max} E, \quad t = 1, \ldots, T, \quad e_n \in \mathbb{E}_n, \quad n = 1, \ldots, N. \tag{10}
\]

The proof of Theorem 1 is given by Appendix A. Let vector \( e = [e_1, \ldots, e_N] \) denote the energy consumption schedule for all consumers. Since (10) is a quadratic programming problem, we have the following expression of (10) as

\[
\min_{e \in \mathbb{Z}} \frac{1}{2} e'Qe \\
\text{s.t.} \quad Ae \leq b,
\]

where \( Q = \begin{pmatrix} a_1^t & 0 & \ldots & 0 \\
0 & a_2^t & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & a_T^t \end{pmatrix}_{\mathbb{T} \times \mathbb{T}} \) is a matrix, \( A = \begin{pmatrix} I_{1 \times \mathbb{N}_M} & 0_{1 \times \mathbb{N}_M} & \cdots & 0_{1 \times \mathbb{N}_M} \\
0_{1 \times \mathbb{N}_M} & I_{1 \times \mathbb{N}_M} & \cdots & 0_{1 \times \mathbb{N}_M} \\
\vdots & \vdots & \ddots & \vdots \\
0_{1 \times \mathbb{N}_M} & 0_{1 \times \mathbb{N}_M} & \cdots & I_{1 \times \mathbb{N}_M} \end{pmatrix}_{\mathbb{T} \times \mathbb{T}} \) and \( b = (\Gamma_{\max} E)_{1 \times \mathbb{M}} \). The dual function of (11) is

\[
q(\mu) = \inf_{e \in \mathbb{Z}} \left\{ \frac{1}{2} e'Qe + \mu'(Ae - b) \right\}. \tag{12}
\]

where \( \mu = [\mu_1, \ldots, \mu_T]^t \) is a \( T \times 1 \) vector. The infimum is attained for \( \hat{e} = -Q^{-1}A\mu \), which should be checked whether \( \hat{e} \in \mathbb{E} \) is satisfied. In case of \( \hat{e} \notin \mathbb{E} \), the exhausted method is used to find infimum \( \hat{e} \in \mathbb{E} \) (We search the values of \( \hat{e} \) down to the second decimal point which may suggest the convinced precision). A straightforward calculation after substituting \( \hat{e} \) in (12), yields

\[
q(\mu) = \frac{1}{2} \mu'\tilde{A}^{-1}A'\mu - \mu'b. \tag{13}
\]

According to the duality theory [34], the dual problem can be written as

\[
\min_{\mu} \frac{1}{2} \mu'\tilde{P}\mu + b'\mu \\
\text{s.t.} \quad \mu \geq 0,
\]

where \( \tilde{P} = AQ^{-1}A' \). Note that the dual problem is also a quadratic program, but it has simpler constraints than the primal problem (P1). It is easy to solve (13) in a centralized fashion using convex programming techniques [34] [35]. Since the primal problem is a convex quadratic program which has a finite optimal value, both the primal and the dual programs have an optimal solution and their optimal values are equal [34]. If \( \mu^* \) is the dual optimal solution, let \( e^* \) denote the optimal solution of the primal problem (P1), then the following holds,

\[
e^* = -Q^{-1}A\mu^*. \tag{14}
\]

**C. Distributed Energy Scheduling Algorithm**

In the previous section, the proposed energy cost minimization problem (P1) can be solved by the RS in a centralized fashion. In order to do global optimization, the RS needs to know the energy consumption information of all appliances of each consumer. Hence, in order to preserve the consumer’s privacy, we propose a distributed approach in solving (P1). We solve the problem at the level of the smart meter in each consumer’s household without the need to reveal its individual appliance consumption profiles. In addition, we propose that only the total energy consumption value be reported to the RS, minimizing the amount of information exchanged among the smart meters and the RS.

For consumer \( n \), the optimal schedule can be determined by solving the following local optimization problem:

\[
\min_{e_n \in \mathbb{E}_n} \frac{1}{2} e_n'Q_n e_n \\
\text{s.t.} \quad A_ne_n \leq b_n, \tag{14}
\]

where \( Q_n = \begin{pmatrix} a_1^t & 0 & \ldots & 0 \\
0 & a_2^t & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & a_T^t \end{pmatrix}_{\mathbb{T} \times \mathbb{T}} \) is a matrix, \( A_n = \begin{pmatrix} I_{1 \times \mathbb{M}} & 0_{1 \times \mathbb{M}} & \cdots & 0_{1 \times \mathbb{M}} \\
0_{1 \times \mathbb{M}} & I_{1 \times \mathbb{M}} & \cdots & 0_{1 \times \mathbb{M}} \\
\vdots & \vdots & \ddots & \vdots \\
0_{1 \times \mathbb{M}} & 0_{1 \times \mathbb{M}} & \cdots & I_{1 \times \mathbb{M}} \end{pmatrix}_{\mathbb{T} \times \mathbb{T}} \) is the number of the appliances of each consumer and \( b_n = (\Gamma_{\max} E)_{1 \times \mathbb{M}} \). The dual function of (14) is

\[
q_n(\mu) = \inf_{e_n \in \mathbb{E}_n} \left\{ \frac{1}{2} e_n'Q_n e_n + \mu'(A_ne_n - b_n) \right\}. \tag{15}
\]
Algorithm 1: Distributed algorithm executed by RS and nth consumer

1: **Repeat**;
2: **At random time instances** **Do**;
3: **Randomly initializes** $\mu = [\mu_1, \ldots, \mu_T]$;
4: Broadcasts $\mu$ to all consumers;
5: **If** the load value $\{L_{n,t}\}_{t=1}^T$ from nth consumer is received 
   **Then**;
6: Calculates $P$ according to $P = AQ^{-1}A'$;
7: Updates $\mu$ according to (16);
8: **End**;
9: **At the residential smart meters**
10: **If** vector $\mu$ is received 
   **Then**;
11: Calculates the values of $e_n$ according to $e_n = -Q_n^{-1}A_n\mu$;
12: **Updates** $e_n$ according to the new value;
13: Calculates $\{L_{n,t}\}_{t=1}^T$ according to $e_n$ and sends the value to the RS;
14: **End**;
15: **End**;
16: **End**;
17: **Until** no consumer sends any new schedule to the RS.

where $\mu$ is a $T \times 1$ vector. Both the local primal problem (14) and the local dual problem (15) have an optimal solution and their optimal values are equal [34]. Let $e^*_n$ denote the optimal solution of the local primal problem (14), then the following holds,

$$e^*_n = -Q^{-1}_n A_n \mu^*.$$

To obtain the feasible solution, we use the same constraint of $\Gamma_{\text{PAR}}$ in the local optimization problem as in the global optimization problem ($P1$). If the energy consumption of each consumer individually satisfies PAR constraint then it also ensures that global PAR constraint is also satisfied. Next, we introduce the update process of $\mu$ at RS when $e_n$ is received from user $n$. The update process of $\mu$ at RS can guarantee the global $\Gamma_{\text{PAR}}$ constraint is still satisfied in the proposed distributed algorithm. Let $\mu_t$ and $b_t$ denote the tth column of $\mu'$ and $b'$, respectively. Let $p_{tt}$ denote the tth diagonal element of $P$, where $P = AQ^{-1}A'$. We have the following iteration (the derivation is presented in the Appendix B),

$$\mu_t \triangleq \max \left\{0, \mu_t - \frac{1}{p_{tt}} (b_t + \sum_{k=1}^T p_{tk} \mu_k) \right\}. (16)$$

We propose Algorithm 1 to solve problem (14) and (16) at consumers and RS respectively, in a distributed fashion. The related information exchange between the RS and each consumer are illustrated in Fig. 3. At the RS, the algorithm starts with some random initial conditions, i.e., the RS assumes a random vector $\mu$. This assumption is required since at the beginning, the RS has no prior information about the consumers. In line 4, the RS broadcasts the value of the $\mu$ to all residential smart meters. The loop in line 5 to 8 is for the RS to update the $\mu$ once $e_n$ from the nth consumer is received. At the residential smart meter, if $\mu$ is received, each smart meter individually calculates its own version of local energy consumption scheduling vector $e_n$ in Line 10. The consumers locally solve the local schedule according to the new announced $\mu$. Then, each consumer updates the RS with its new value of $e_n$, which is the total local power consumption per time interval. In this way, residences do not need to reveal their individual appliance consumption profiles. Finally, the loop in Lines 1 to 17 is executed until the algorithm converges.

**D. Convergence**

The distributed Algorithm 1 presented in the previous subsection allows consumers coordinate with RS to obtain the optimal energy scheduling. In this subsection, we will prove the convergence of the proposed distributed algorithm to illustrate its usability. The assessment is based on the following theorem.

**Theorem 2.** RS starts from any randomly selected initial conditions. Algorithm 1 converges to its fixed point, if the broadcast of the iteration vector from RS and the updates of the individual energy consumption scheduling vectors from consumers are accomplished successfully*.

**Proof:** After successfully obtaining the iteration vector for each consumer, the optimal energy schedule can be achieved by solving the optimization problem (14). From Algorithm 1, the energy cost in the system either decreases or remains unchanged every time a consumer updates its energy consumption schedule. Since the energy cost is bounded below (e.g., the energy cost is always nonnegative), the convergence to some fixed point is evident. On the other hand, at the fixed point of Algorithm 1, no consumer can reduce its cost by deviating from the fixed point when operating at its optimal energy schedule. This directly indicates that the fixed point is the optimal point among all the consumers.

The proposed scheduling scheme is usable by operators to reduce peak loads through multi-tier electricity pricing. In this case, consumers are able to reduce their costs by scheduling the operation period and power of the residential appliances. However, this scheduling process ignores the consumer’s usage preferences for both shiftable and throttleable appliances which may not be suitable in a practical residential environment. The results from this optimization should then be used to serve as a baseline for comparison purposes.

*The complete information exchanging between smart meters and RS can be guaranteed by wire or wireless communication technologies [6]-[8]. The energy consumption scheduling for residential network under incomplete information exchanging scenario is beyond the scope of this paper.
IV. Energy Cost Minimization with Consumer’s Preference

In this section, we propose an alternative energy consumption scheduling scheme that considers consumers’ usage preference. In this scheme, we are not only interested in minimizing energy cost, but also consider the preference of the consumer when scheduling household appliances. A multi-objective optimization problem is formulated and firstly solved in a centralized manner. A subgradient method based distributed algorithm is then proposed to solve the minimization problem in a distributed manner.

A. Problem Formulation

For shiftable appliances, we assume that the consumers have a specific required operational interval $[D_{n,s}, U_{n,s}]$, which denotes the range of the operation time interval for the $s$th shiftable appliance of the $n$th consumer. Without loss of generality, we have $D_{n,s} \leq \alpha_{n,s} < \beta_{n,s} \leq U_{n,s}$, where $\alpha_{n,s} \in \mathcal{T}$ and $\beta_{n,s} \in \mathcal{T}$ are the start and end of a time interval that appliance $s$ has been scheduled. Let $e_{n,s}$ denote the energy schedule of the shiftable appliance $s$ in consumer $n$. The operation delay of the $s$th shiftable appliance in consumer $n$ is denoted by

$$O_{n,s}(e_{n,s}) = \arg \min_{t} \{e_{n,s}^t > 0\} - D_{n,s}.$$  

Obviously, the operation delay should be minimized as well as the energy cost of the shiftable appliance.

For the throttleable appliances, the power gap of the $r$th throttleable appliance for $n$th consumer is denoted as $G_{n,r}$, which reflects the difference between the consumer’s preferred power $P_{n,r}^{pre}$ and the scheduled power $e_{n,r}$, as adjusted by the RS. We then have

$$G_{n,r}(e_{n,r}) = \sum_{t=1}^{T} (P_{n,r}^{pre} - e_{n,r}^t).$$

To maximize a consumer’s comfort and satisfaction, the power gap also needs to be minimized.

We can therefore incorporate the operation delay, energy gap and energy cost minimization into an optimization problem to find an optimal scheduling vector $e_n = \{e_{n,i}, e_{n,s}, e_{n,r}\}, n \in \mathcal{N}$. An energy consumption schedule of appliances can be formulated in terms of minimizing the energy costs, operation delay and the energy gap of all consumers, which can be expressed as the following optimization problem:

$$(P2) \min_{e_n} \Omega_1 \sum_{t=1}^{T} \mathcal{R}(L_t) + \Omega_2 \sum_{n \in \mathcal{N}} \sum_{s \in \mathcal{S}_n} O_{n,s}(e_{n,s}) + \Omega_3 \sum_{n \in \mathcal{N}} \sum_{r \in \mathcal{R}_n} G_{n,r}(e_{n,r})$$

s.t. $\Gamma_{PAR} = \frac{\max_{t \in \mathcal{T}} L_t}{\phi} \sum_{t \in \mathcal{T}} L_t \leq \Gamma_{max}$

$$\sum_{t=\alpha_{n,s}}^{\beta_{n,s}} e_{n,s}^t = E_{n,s}, \quad e_{n,s}^t = 0, \quad \forall t \in \mathcal{T} \setminus \mathcal{T}_{n,s},$$

$$\sum_{t=\alpha_{n,r}}^{\beta_{n,r}} e_{n,r}^t \leq E_{n,r}, \quad e_{n,r}^t = 0, \quad \forall t \in \mathcal{T} \setminus \mathcal{T}_{n,r},$$

$$0 < e_{n,r}^t \leq P_{n,r}^{max}, \quad \forall t \in \mathcal{T}_{n,r},$$

$$P_{n,r}^{tot} \leq e_{n,r}^t \leq P_{n,r}^{pre}, \quad \forall t \in \mathcal{T}_{n,r},$$

where $e = [e_1, \ldots, e_N]$ denote the energy consumption schedule for all consumers, $L_t = \sum_{n \in \mathcal{N}} \left( \sum_{i \in \mathcal{I}_n} e_{n,i}^t + \sum_{s \in \mathcal{S}_n} e_{n,s}^t \right)$, $\mathcal{T}_{n,s} = \{D_{n,s}, \ldots, U_{n,s}\}$, $\mathcal{T}_{n,r} = \{\alpha_{n,r}, \ldots, \beta_{n,r}\}$ where $e_{n,r} \in \mathcal{T}$ and $\beta_{n,r} \in \mathcal{T}$ are the fixed start and end times of throttleable appliances. $\Omega_1, \Omega_2$ and $\Omega_3$ are the weight which can be adjusted based on consumers’ preference. Given $\Omega_1, \Omega_2, \Omega_3$, and following Theorem 1 as described in Section III-B, we have simplification of (P2) as

$$\min_{e_n} \Omega_1 \sum_{t=1}^{T} \mathcal{R}(L_t) + \Omega_2 \sum_{n \in \mathcal{N}} \sum_{s \in \mathcal{S}_n} O_{n,s}(e_{n,s}) + \Omega_3 \sum_{n \in \mathcal{N}} \sum_{r \in \mathcal{R}_n} G_{n,r}(e_{n,r})$$

s.t. $L_t \leq \Gamma_{max} E, \quad t = 1, \ldots, T,$

$$e_{n} \in \mathcal{E}_n, \quad n = 1, \ldots, N.$$  

B. The Dual of Energy Minimization Problem

Let $\{\phi_t, t = 1, \ldots, T\}$ denote the Lagrange multiplier of problem (17), the Lagrangian function for (17) is

$$L(\phi, e_n) = \Omega_1 \sum_{t=1}^{T} \mathcal{R}(L_t) + \Omega_2 \sum_{n \in \mathcal{N}} \sum_{s \in \mathcal{S}_n} O_{n,s}(e_{n,s})$$

$$+ \Omega_3 \sum_{n \in \mathcal{N}} \sum_{r \in \mathcal{R}_n} G_{n,r}(e_{n,r}) + \sum_{t=1}^{T} \phi_t (L_t - \Gamma_{max}).$$

The dual function and the dual problem take the form

$$q(\phi) = \min_{e_n \in \mathcal{E}_n} L(\phi, e_n).$$

The Lagrange multiplier $\phi_t$ can be interpreted as the price charged by the utility company at $t$ if the consumed energy for all consumers exceed the constraint $\Gamma_{max} E$. Hence, the units of Lagrange multipliers can be interpreted to be monetary units/kWh. Moreover, the dual problem can be written as

$$q^* = \max_{\phi \geq 0} q(\phi).$$
Note that (20) is a convex problem. It can be solved in a centralized manner by RS. Because problem (17) is convex and has zero duality gap, let $e^*$ denote the optimal solution of (17), and $\phi^*$ denote the optimal Lagrange multipliers corresponding to the dual solution [34]. Variables $e^*$ are also the minimizers in (19) for $\phi = \phi^*$, i.e., $q(\phi^*) = L(\phi^*, e^*)$.

For the same reasons as the previous section, we then develop a distributed algorithm for that can solve $(P2)$ in a distributed manner. The optimization problems are assigned to the smart meters of individual consumers. These meters are then, coordinated through the RS to obtain the joint optimal schedule. To achieve this, the distributed iterations are described in Section IV-C, and their convergence is established in Section IV-D.

### C. Distributed Energy Scheduling Algorithm with Consumer’s Preference

In this subsection, a distributed energy scheduling algorithm considering consumers’ preferences is proposed. For any consumer $n \in \mathcal{N}$, the optimal schedule can be determined by solving the following local optimization problem:

$$
e_n(l) \in \arg \min_{e_n \in \mathcal{E}_n} \sum_{t=1}^{T} \mathcal{R}(L_{n,t}) + \sum_{t=1}^{T} \phi_t(l)(L_{n,t} - \Gamma_{\max}E) + \Omega_2 \sum_{s \in S_n} O_{n,s}(e_{n,s}) + \Omega_3 \sum_{r \in R_n} G_{n,r}(e_{n,r}), \quad n = 1, 2, \cdots, N,
$$

(21)

Then, we develop a subgradient method to solve the local scheduling problem (21). The iteration utilizes the Lagrange minimization function (19) from the dual problem and minimizes with respect to $e_n$ for each consumer $n$. Specifically, the subgradient method consists of the following iterations, indexed by $l = 1, 2, \cdots$ and initialized with arbitrary $\phi_t(l) \geq 0$:

$$
\phi_t(l + 1) = \left\{ \phi_t(l) + \lambda_t \left[ (L_h(l) - \Gamma_{\max}E) \right] \right\}^+, \quad t = 1, 2, \cdots, T,
$$

(22)

where $\lambda_t$ is the step size and $\{\cdot\}^+ = \max\{0, \cdot\}$.

Then, we propose distributed Algorithm 2 to solve problem (21) and (22) at the RS and consumers, respectively. The flow of the algorithm is as follows. Similar to Section III, the RS takes responsibility for the updating and broadcasting Lagrange multipliers $\{\phi_t(l)\}_{l=1}^{T}$ to all the consumer’s smart meters. At each consumer, upon receiving the Lagrange multipliers $\{\phi_t(l)\}_{l=1}^{T}$, the associated smart meter solves its own version of local optimization problem (21). The related information exchange between RS and each consumer are illustrated in Fig. 4. Finally, the loop in Lines 1 to 16 is executed until the algorithm converges.

### D. Convergence

In this subsection, we will prove the convergence of Algorithm 2. According to Theorem 2, i.e., the broadcast of the iteration vector from the RS and the updates of the individual energy consumption scheduling vectors from the consumers are successful. We note that the subgradient method will work if a sufficiently small step size $\beta$ is used. This shown by the following theorem.

**Theorem 3.** The proposed Algorithm 2 will converge if every $\phi_t(l)$ and dual optimal solution $\phi^*$ have

$$
\| \phi_t(l + 1) - \phi^* \| \leq \| \phi_t(l) - \phi^* \|,
$$

for all step size $\lambda_t$ such that

$$
0 < \lambda_t < \frac{2q(\phi^*) - q(\phi_t(l))}{\| L_h(l) - \Gamma_{\max}E \|^2}
$$

(23)

The proof of Theorem 3 is given in Appendix C. Theorem 3 can be used to establish convergence and rate of convergence results for the subgradient method with step size rules satisfying (23). However, unless we know the dual optimal value $q(\phi^*)$, which is rare, the range of step size (23) is unknown. In this paper, we use the step size formula

$$
\lambda_t = \frac{\alpha \eta(q_t(l) - q(\phi_t(l)))}{\| L_h(l) - \Gamma_{\max}E \|^2}
$$

where $q_t(l)$ is an approximation to the optimal dual value and $0 < \alpha < 2$. To estimate the optimal dual value $q_t(l)$, we can use the best current dual value

$$
\hat{q}_t(l) = \max_{0 \leq i \leq T} q_t(\phi(i))
$$
as an upper bound. Meanwhile, any primal value corresponding to a primal feasible solution can be used. The primal feasible solutions are naturally obtained in the course of the Algorithm 2 if the energy scheduling vector from all consumers are achieved successfully. Here, we employ the following way to choose \( \alpha_l \) and \( q_l(l) \) in the step size formula: \( \alpha_l = 1 \) for all \( l \) and \( q(l) \) is given by

\[
q(l) = \hat{q}_I(l) + \delta(l),
\]

where \( \hat{q}_I(l) \) is the best current dual value and \( \delta(l) \) is a positive number, which is increased by a certain factor if the previous iteration was a “success”, i.e., it proved the best current dual value, and is decreased by some other factor otherwise. Thus, \( \delta(l) \) may be viewed as an “aspiration level” of improvement that we hope to attain in subsequent iterations. If upper bounds \( \hat{q}_I(l) \) to the optimal dual value are available, then a natural improvement to the above equation is

\[
q(l) = \min \{ \hat{q}_I(l), \hat{q}_I(l) + \delta(l) \}.
\]

We refer to [36] for related convergence analysis. We use a variation of the method, is to choose

\[
q(l) = (1 + \psi(l))\hat{q}_I(l),
\]

where \( \psi(l) \) is the number greater than zero, which is increased or decreased based on algorithm progress. This variation requires that \( \hat{q}_I(l) > 0 \).

**Discussion:** In this paper, we considered scheduling schemes that explored combinations of two design parameters - whether the scheduling scheme considered consumer preferences and whether the scheduling should be centralized or distributed. Centralized and distributed scheduling schemes which do not consider consumer preferences are Centralized-Nonprefer and Distributed-Nonprefer respectively. Similarly, centralized and distributed scheduling schemes which consider consumer preferences are named as Centralized-Prefer and Distributed-Prefer, respectively. Table I shows the information exchanged between RS and consumer \( n \) in these four proposed scheduling schemes. It can be seen that the information exchange in centralized scheduling schemes are more than that in the distributed scheduling schemes. This is because the distributed schemes only need the total energy consumption of each consumer instead of the detailed individual appliance consumption information of all consumers in the centralized schemes. More importantly, this implies that the privacy of the consumer’s energy usage is carefully protected in distributed schemes.

### Table I

<table>
<thead>
<tr>
<th>Scheduling Type</th>
<th>Centralized</th>
<th>Distributed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nonprefer</td>
<td>{(e_{n,t}^{i,n}, \beta_{n,t}^{i,n}, e_{n,t}^{p,r})}<em>{t=1}^{T}, \mu, {L_n(t)}</em>{t=1}^{T} }</td>
<td>{(e_{n,t}^{i,n}, \beta_{n,t}^{i,n}, e_{n,t}^{p,r})}<em>{t=1}^{T}, {(\phi</em>{l}(l))}_{t=1}^{T} }</td>
</tr>
<tr>
<td>Prefer</td>
<td>{(e_{n,t}^{i,n}, \beta_{n,t}^{i,n}, e_{n,t}^{p,r})}<em>{t=1}^{T}, {(\phi</em>{l}(l))}_{t=1}^{T} }</td>
<td>{(e_{n,t}^{i,n}, \beta_{n,t}^{i,n}, e_{n,t}^{p,r})}<em>{t=1}^{T}, {(L_n(t))}</em>{t=1}^{T} }</td>
</tr>
</tbody>
</table>

### V. Simulation Results

In our considered residential smart grid system, we set a optimization period as 24 hours (i.e. \( T = 24 \)) and the length of each time interval \( t \in T \) is 1 hour. We assume that each consumer (home) has between 10 to 20 essential appliances with strict energy consumption scheduling constraints. Such appliances may include electric stoves (daily usage \( E: 1.89 \) kWh for self-cleaning and 2.01 kWh for regular) and lighting (daily usage for 10 standard bulbs \( E: 1.00 \) kWh) [26]. Each home also has between 10 to 20 appliances with shiftable and throttleable operations. These are the appliances that the smart meter can schedule due to soft energy consumption scheduling constraints. The shiftable appliances may include dishwashers (daily usage \( E: 1.44 \) kWh, \([D, U] = [18, 22]\)), washing machines (daily usage \( E: 1.49 \) kWh for energy-start, 1.94 kWh for regular, \([D, U] = [18, 24]\)), clothes dryers (daily usage \( E: 2.50 \) kWh, \([D, U] = [1, 8]\)), and PHEVs (daily usage \( E: 9.9 \) kWh, \([D, U] = [1, 8]\)) [26]. The throttleable appliances may include the air-conditioning unit (with a preferred and tolerance demands of 750W and 600W, respectively) and the refrigerator (with preferred and tolerance demands of 140W and 112W respectively). In addition, for a comparison with [26], we adopt the same pricing model. The parameters for the pricing scheme are \( b_t = c = 0 \) for all \( t \in T \) and \( a_t = 0.3 \) cents during daytime hours, i.e., from 8:00 in the morning to 12:00 at night and \( a_t = 0.2 \) cents during the night, i.e., from 12:00 at night to 8:00 AM the day after.

#### A. Performance Analysis for Different Scheduling Schemes

In this subsection, we seek to compare the performances of the different scheduling schemes. The common comparator is the energy cost involved in each of the scheme. In addition to the four defined schemes from the section before, a baseline scheme (no-scheduling) where no scheduling of appliances is carried out and the scheme proposed in [26]. Automatic Demand-Side Management (ADM) are compared. As a summary, the four previously defined schemes are Centralized-Nonprefer, Distributed-Nonprefer, Centralized-prefer and Distributed-prefer. In the Prefer cases, we set the weights for the consumer’s usage preference as \( \Omega_1 = 1, \Omega_2 = 600, \Omega_3 = 1 \). We will evaluate the performance of the proposed scheduling scheme under different combinations of the weights in the next subsection.

Fig. 5 and Fig. 6 show the energy costs in the different schemes with \( \Gamma_{max} = 2 \) and \( \Gamma_{max} = 4 \) respectively. It is shown that the energy costs achieved by the proposed schemes (Centralized-Nonprefer and Centralized-Prefer) are less than those of the baseline schemes of no-scheduling and ADM [26]. It is obvious that there should be cost savings when we adopt scheduling strategies for appliances. Those costs savings advantage over ADM is attributable to the fact that the proposed schemes are not only able to schedule the shiftable appliances to the low electricity price periods, but also reduce the power draw of the throttleable appliances while the price is high. We also note that the energy cost in the Centralized-Prefer case is higher than that in the Centralized-Nonprefer case. That is, the additional objectives involved in
the Centralized-Prefer case results in the reduction of cost savings to improve consumers’ satisfaction through reduction in controlling of shiftable and throttleable appliances during high price periods. This translates to shorter operation delays and lower energy gaps than that in the Centralized-Nonprefer scheme as shown in Fig. 7 and Fig. 8, respectively. Moreover, we observe that the energy costs in the distributed scheduling schemes are close but higher than that in the centralized scheduling cases. That is, the distributed schemes only use the local information to obtain the optimal energy schedule which may lead to the suboptimal schedule for each of the consumer. In contrast, centralized schemes are able to achieve optimal scheduling of energy consumption through the knowledge of global information.

In addition, Fig. 5 and Fig. 6 show that the energy cost in \( \Gamma_{max} = 2 \) scenario is higher than that in \( \Gamma_{max} = 4 \) scenario. This is because a lower \( \Gamma_{max} \) indicates the stricter constraint of energy consumption allowable for consumers in each hour. In this case, some cost reduction driven operations in high price periods may be unable to shift to lower periods due to a lower peak allowed for electricity consumption.

Next, we show the resulting PAR and daily cost for 10 consumers for all compared schemes in Table II. The proposed centralized schemes can achieve a lower cost and lower PAR than the ADM scheme. That is, the centralized scheme can obtain better scheduling performance by knowing the global information of all users. Then, for the proposed distributed schemes, we can see that the distributed-nonprefer scheme has the same PAR as the ADM scheme (but lower cost) while the distributed-prefer scheme has slightly higher PAR as compared to ADM. This indicates that the proposed distributed scheme without considering the user’s preference can obtain better performance as ADM (same PAR but lower cost). In the case when the user’s preference is considered, the peak loads may increase which leads to the higher PAR.

**TABLE II**

<table>
<thead>
<tr>
<th>Schemes</th>
<th>PAR</th>
<th>Daily Cost (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No-scheduling</td>
<td>2.7</td>
<td>63.55</td>
</tr>
<tr>
<td>ADM [26]</td>
<td>2.4</td>
<td>50.83</td>
</tr>
<tr>
<td>Centralized-Prefer</td>
<td>2.2</td>
<td>42.47</td>
</tr>
<tr>
<td>Centralized-NonPrefer</td>
<td>2.1</td>
<td>34.05</td>
</tr>
<tr>
<td>Distributed-Prefer</td>
<td>2.5</td>
<td>44.39</td>
</tr>
<tr>
<td>Distributed-NonPrefer</td>
<td>2.4</td>
<td>38.72</td>
</tr>
</tbody>
</table>
B. Performance Analysis with Scheduling Weight

When considering schedules that account for user preferences, the performance of the appliance schedule will be influenced by the different combination of the weights \((\Omega_1, \Omega_2, \Omega_3)\). Given \(\Gamma_{\text{max}} = 4\), to illustrate the influences, we define four weight combinations as: balanced criteria \((\Omega_1 = 1, \Omega_2 = 1, \Omega_3 = 1)\), cost-sensitive criteria \((\Omega_1 = 600, \Omega_2 = 1, \Omega_3 = 1)\), delay-sensitive criteria \((\Omega_1 = 1, \Omega_2 = 600, \Omega_3 = 1)\) and gap-sensitive criteria \((\Omega_1 = 1, \Omega_2 = 1, \Omega_3 = 600)\).

Fig. 9 shows the energy cost in terms of the different weighting criteria of consumer preferences. Note that the energy cost under the balanced criteria and cost-sensitive criteria are nearly the same. This indicates that the \(\Omega_1\) is dominant of energy cost in (17). However, in the delay-sensitive and gap-sensitive cases, the energy cost is higher than that in the former two cases. It is important to note that the consumer will incur higher energy costs than balanced and cost-sensitive criteria if they emphasize the need to minimize the operation delay (with larger \(\Omega_2\)) or to reduce the energy gap (with larger \(\Omega_3\)). Also, we observe that the energy cost in the delay-sensitive criteria is higher than that in the gap-sensitive criteria. This is because the shiftable appliances has higher daily energy usage than that of the throttleable appliances in our simulation settings. Hence, when consumers focus on minimizing the operation delay of the shiftable appliances, more energy cost will be caused than that in the gap-sensitive case in which consumers devote to reduce the energy gap of the throttleable appliances.

The operation delay and energy gap in terms of different weight criteria are shown in Fig. 10 and Fig. 11, respectively. In Fig. 10, the operation delay has the lowest value in the delay-sensitive criteria when that delay minimization is the dominant criterion in the multiple objective optimization problem (17). Similarly, in Fig. 11, the energy gap can achieve the minimum value under the gap-sensitive criteria (17).

C. Convergence of Distributed Algorithms

The total energy costs while Algorithm 1 and Algorithm 2 proceed along their distributed iterations are shown in Fig. 12 and Fig. 13, respectively. In Fig. 12, the distributed iterations are operated in terms of different values of \(\Gamma_{\text{max}}\), where \(\Gamma_{\text{max}} = 2\) and \(\Gamma_{\text{max}} = 4\). We observe that when Algorithm 1 is running, the energy cost decreases until the algorithm converges after about 22 iterations. Similar trends can be revealed as the consumers and RS jointly run Algorithm 2 as shown in Fig. 13. Also, in both of the \(\Gamma_{\text{max}} = 2\) and \(\Gamma_{\text{max}} = 4\) cases, it is seen that the proposed distributed Algorithm 2 converges quickly. Steady state is also reached after around 22 iterations when the energy cost is minimized.

VI. Conclusion and Future Work

With the constraint of PAR, we presented a demand-side management system based on energy consumption scheduling...
for the residential smart grid. By scheduling of shiftable appliances and adjusting the power of throttleable appliances, the residential scheduler is able to minimize the energy cost according to a multi-tier and time-varying pricing structure. Taking the consumer’s preferences into account, we also extended our scheduling scheme to consider instances where the shiftable appliances and the throttleable appliances have specific operational periods and preferred power consumption levels. It can also be applied by the consumer to balance the tradeoff between appliance usage preferences and economic benefits. Moreover, distributed algorithms are proposed to solve the energy cost minimization problem. By requiring limited message exchanges between consumers and residential scheduler, the proposed distributed algorithms do not need to reveal the consumers’ energy consumption profile, which is important in protecting the privacy of consumers.

In the future, we plan to extend our work in several directions. First, we can design dynamic pricing or other incentive schemes that can better manage the demand response. Second, our model system can be extended to scenarios where the shiftable appliances and the throttleable appliances have specific operational periods and preferred power consumption levels. It can also be applied by the consumer to balance the tradeoff between appliance usage preferences and economic benefits. Moreover, distributed algorithms are proposed to solve the energy cost minimization problem. By requiring limited message exchanges between consumers and residential scheduler, the proposed distributed algorithms do not need to reveal the consumers’ energy consumption profile, which is important in protecting the privacy of consumers.

APPENDIX A

PROOF OF THEOREM 1

Theorem 1. If \( L_t = \sum_{e_n \in N} (\sum_{i \in T_n} e_{n,i}^t + \sum_{s \in S_n} e_{n,s}^t + \sum_{r \in R_n} e_{n,r}^t) \) and \( E = \frac{1}{T} (E_1 + \sum_{e_n \in N} \sum_{s \in S_n} E_{n,s} + T \sum_{e_n \in N} \sum_{r \in R_n} p_{n,r}) \), the problem \((P1)\) can be rewritten as

\[
\min_{e_n} \sum_{t=1}^{T} R(L_t)
\quad s.t. \quad L_t \leq T \Gamma_{max} E, \quad t = 1, \ldots, T,
\quad e_n \in \mathbb{E}_n, \quad n = 1, \ldots, N.
\]

Proof: Since \( L_t = \sum_{e_n \in N} (\sum_{i \in T_n} e_{n,i}^t + \sum_{s \in S_n} e_{n,s}^t + \sum_{r \in R_n} e_{n,r}^t) \), the last constraint in \((P1)\) can be written as

\[
\max_{t \in T} L_t \leq \Gamma_{max} T \frac{1}{T} \sum_{t \in T} L_t
\]

\[
= \frac{1}{T} \sum_{t \in T} \sum_{e_n \in N} \sum_{i \in T_n} e_{n,i}^t + \sum_{s \in S_n} e_{n,s}^t + \sum_{r \in R_n} e_{n,r}^t.
\]

We observe that the following relations hold,

\[
\sum_{t \in T} \sum_{e_n \in N} \sum_{i \in T_n} e_{n,i}^t = E_t,
\]

\[
\sum_{t \in T} \sum_{e_n \in N} \sum_{s \in S_n} e_{n,s}^t \leq \sum_{n \in N} \sum_{s \in S_n} E_{n,s},
\]

\[
\sum_{t \in T} \sum_{e_n \in N} \sum_{r \in R_n} e_{n,r}^t \leq T \sum_{n \in N} \sum_{r \in R_n} p_{n,r}^{pre}.
\]

Defining \( E \) as \( E = \frac{1}{T} (E_1 + \sum_{n \in N} \sum_{s \in S_n} E_{n,s} + T \sum_{n \in N} \sum_{r \in R_n} p_{n,r}^{pre}) \), we have

\[
\max_{t \in T} L_t \leq \Gamma_{max} E.
\]

Moreover, it is obvious that the following inequality holds:

\[
L_t \leq \max_{t \in T} L_t \leq \Gamma_{max} E, \quad t = 1, \ldots, T.
\]

Finally, by substituting the inequality in (24), the result follows. \( \blacksquare \)

APPENDIX B

DERIVATION OF THE \( \mu \) ITERATION

Let \( x_t \) denote the \( t \)th column of \( A' \). Since \( Q \) is symmetric and positive definite, the \( t \)th diagonal element of \( P \), given by \( p_{tt} = x_t^T Q^{-1} x_t \), is positive. This means that for every \( t \), the dual function is strictly convex along the \( t \)th coordinate. Therefore, the strict convexity is satisfied and it is possible to use the coordinate ascent method [36]. Because the dual function is quadratic, the minimization with respect to \( \mu \) along each coordinate can be done analytically, and the iteration can be written explicitly.

The first partial derivation of the dual function with respect to \( \mu_t \) is given by

\[
b_t + \sum_{k=1}^{T} p_{tk} \mu_k,
\]

where \( p_{tk} \) are the corresponding elements of the matrix \( P \). Setting the derivative to zero, we see that the unconstrained minimum of the dual along the \( t \)th coordinate starting from \( \mu \) is attained at \( \tilde{\mu}_t \) given by

\[
\tilde{\mu}_t = -\frac{1}{p_{tt}} \left( b_t + \sum_{k \neq t} p_{tk} \mu_k \right) = \mu_t - \frac{1}{p_{tt}} \left( b_t + \sum_{k=1}^{T} p_{tk} \mu_k \right).
\]

Taking into account the nonnegativity constraint \( \mu_t \geq 0 \), we see that the \( t \)th coordinate update has the form

\[
\mu_t \triangleq \max \{0, \tilde{\mu}_t\} = \max \left\{ 0, \mu_t - \frac{1}{p_{tt}} \left( b_t + \sum_{k=1}^{T} p_{tk} \mu_k \right) \right\}.
\]
APPENDIX C

PROOF OF THEOREM 3

Theorem 3. The proposed Algorithm 2 will converge if every $\phi_t(l)$ and dual optimal solution $\phi^*$ have
$$\|\phi_t(l + 1) - \phi^*\| < \|\phi_t(l) - \phi^*\|,$$
for all step size $\lambda_t$ such that
$$0 < \lambda_t < \frac{2g(\phi^*) - q(\phi_t(l))}{\|l_t(l) - \Gamma_{max}E\|^2}.$$

Proof: Let $g_t = L_t(l) - \Gamma_{max}E$, we have
$$\|\phi_t(l) + \lambda_t g_t - \phi^*\|^2 = \|\phi_t(l) - \phi^*\|^2 - 2\lambda_t(\phi^* - \phi_t(l))'g_t + (\lambda_t)^2 \|g_t\|^2,$$
and by using the subgradient inequality,
$$(\phi^* - \phi_t(l))'g_t \geq g(\phi^*) - q(\phi_t(l)),$$
we obtain
$$\|\phi_t(l) + \lambda_t g_t - \phi^*\|^2 \leq \|\phi_t(l) - \phi^*\|^2 - 2\lambda_t(q(\phi^*) - q(\phi_t(l)))'g_t + (\lambda_t)^2 \|g_t\|^2,$$
We can verify that for the range of step size of (25) the sum of last two terms in the above relation is negative. In particular, with a straightforward calculation, we can write this relation as
$$\|\phi_t(l) + \lambda_t g_t - \phi^*\|^2 \leq \|\phi_t(l) - \phi^*\|^2 - \gamma(l)(2 - \gamma(l)) \|q(\phi^*) - q(\phi_t(l))\|^2 \|g_t\|^2,$$
where $\gamma(l) = \frac{\lambda_t \|g_t\|^2}{q(\phi^*) - q(\phi_t(l))}$. If the step size $\lambda_t$ satisfies (25), then $0 < \gamma(l) < 2$, so (26) yields
$$\|\phi_t(l) + \lambda_t g_t - \phi^*\| < \|\phi_t(l) - \phi^*\|.$$ We now note that since $\phi^* > 0$ and the projection operation is non-expansive [36], we have
$$\|\{\phi_t(l) + \lambda_t g_t\}^+ - \phi^*\| \leq \|\phi_t(l) + \lambda_t g_t - \phi^*\|.$$
Because $\phi_t(l + 1) = \{\phi_t(l) + \lambda_t g_t\}^+$, by combining the last two inequalities, the result follows.
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