On the Competition of CDN Companies: Impact of New Telco-CDNs’ Federation

Hyojung Lee, Lingjie Duan, and Yung Yi

Abstract—To cope with consumers’ ever-increasing demand of high-quality contents in the Internet, content providers (e.g., YouTube) mainly pay to global pure-play CDN companies (e.g., Akamai) instead of ISPs for content delivery. This motivates ISPs to offer their own regional CDN services as Telco-CDNs and compete with the pure-play CDNs for CPs in the CDN market. Unlike the pure-play CDNs with global consumer coverage, these Telco-CDNs with global consumer coverage, these Telco-CDNs are also attractive to CPs as they can exploit their own domain knowledge for improving user-perceived QoS and use smarter integration of traffic engineering with more efficient content management (e.g., domain-specific caching). It is expected that Telco-CDNs’ emergence brings about a great change in the existing CDN market, by introducing the competition between new Telco-CDNs with better service quality [5]–[9] and the incumbent pure-play CDNs with larger service coverage [10].

In this paper, we aim at understanding how the new CDN market runs where pure-play and ISP-owned CDNs compete and CPs choose between them, and we allow Telco-CDNs to work independently or federate to different extents (resource-pooling and/or revenue-sharing). We develop a dynamic game theory model to characterize how multiple Telco-CDNs compete with a typical global pure-play CDN (e.g., Akamai) for CPs: in Stage I, Telco-CDNs under a particular federation case and the pure-play CDN decide service pricing to attract CPs; in Stage II, CPs choose between the services provided by Telco-CDNs and the pure-play CDN.

We first consider the traditional case when each Telco-CDN runs the business and separately compete against the pure-play CDN. One can easily conjecture that in such a “separated competition region” each Telco-CDN would have a small market share stemming from its limited consumer coverage, and thus experience dominated competition, leading to a small revenue. This is quantified by our analysis (see Section IV). From Telco-CDNs’ perspective, one of the plausible measures to equip themselves with a bigger market power is to cooperate and form a federation. The idea of federation is indeed considered and seriously discussed in practice, e.g., Cisco’s Open CDN Federation Pilot [11] and IETF’s CDN Interconnection (CDN Interconnection) WG [12]. The federation effects of Telco-CDNs are diverse: (i) ISPs can cut down maintenance cost due to resource pooling and (ii) consumers can still benefit better QoS from Telco-CDNs. Despite these federation advantages, its success largely relies on the existence of federation incentives. In the second part of this paper, we analytically study the economic impact of Telco-CDNs’ federation in a quantitative manner. Depending on the degree of federation in terms of resource pooling and/or revenue sharing, we consider two types of federations: partial and full federations. In the partial federations, Telco-CDNs just cooperate to pool their resource, but in the full federations, they also cooperate in sharing the total revenue as in the cooperative game theory.

The main messages of this paper are in what follows:
o Telco-CDNs’ QoS improvement versus pure-play CDN’s global consumer coverage: they are two key factors to determine the competition outcome between Telco-CDNs and the pure-play CDN for CPs. At the equilibrium, the factors bring about two interesting behaviors of the CDN market: (i) the perfect competition and (ii) the market domination by one CDN type. Both types of providers (Telco-CDNs and pure-play CDN) could not make profits whenever they have equivalent market powers in terms of QoS and consumer coverage due to the perfect competition. Moreover, a CDN type dominates another in attracting most CPs if it has a much larger market power than the competitor.

o Not always a Telco-CDN could increase its revenue by a federation. By the federation, Telco-CDNs can enlarge their consumer coverages and thus increase the market power against the pure-play CDN. However, this may not lead to the increase of each or even any Telco-CDN’s revenue due to increase of competition or the threat of perfect competition with the pure-play CDN. A Telco-CDN in high coverage regime can always increase its revenue whereas a Telco-CDN in low coverage regime can conditionally increase its revenue, where the conditions are to avoid the perfect competition.

o Under a centralizer’s coordination, Telco-CDNs get more freedom from the threat of perfect competition. Since a Telco-CDN having incentive to federate can economically help another who does not have incentive, by revenue sharing, the existence of centralizer would encourage the full federation and increase the viability of Telco-CDNs.

II. RELATED WORK

We first introduce the related works on the economic interaction between ISPs, CPs and CDNs. Hau and Brenner [13] studied the interaction between ISPs and CDNs by taking a model of the pricing decisions of ISPs and CDNs, and showed that ISPs have relatively high market power and obtain profits from CDNs to compete for end users. Regarding the relation between CDNs and CPs, Hosanagar et al. [14] addressed the questions on the optimal pricing policies of CDNs. They found that for Markovian traffic, CDNs should provide volume discounts to CPs, but when traffic is bursty and CPs have varying levels of traffic burstiness, volume discounts may be suboptimal and may even be replaced by volume taxes. Although CDNs was included in their modelling, the studies in [13], [14] place CDNs as an entity separated from ISPs, whereas our work handles the case when the features from ISPs and CDNs are realized by a single player, i.e., Telco CDNs.

In terms of Telco-CDNs and their federation, there are several studies that technically investigate the performance improvement in the context of reduction of congestion, traffic and latency. Cho et al. [15] proposed a content delivery architecture called ISP centric Content Delivery (iCODE) by which an ISP can provide content delivery services as well. They showed that compared with CDN and P2P systems, iCODE can offer reduced delivery latency by reducing inter-ISP traffic and allowing traffic engineering. Moreover, Nam and Park [16] studied the performance of Telco-CDNs’ federation when applied to the fast-growing cellular Internet traffic and they showed Telco-CDN federation can reduce 16.2% to 29% of IXP traffic. Also, in terms of ISP and CP cooperation, Jiang et al. [7] showed that when ISPs and CPs jointly optimize the process of server selection and traffic engineering with main focus on the technical aspects, i.e., reduction of congestion and latency via ISP-CDN cooperation. To the best of our knowledge, our work is the first study on network economics in terms of Telco-CDNs’ competitiveness under different types of federation and different revenue sharing policies. Lee. et al [17] also studied the economic impact of Telco-CDN, they focused on the trade-off between QoS of Telco-CDNs and the disutility of their federation, but they did not focus on the types of federation. Our two-stage dynamic game model characterizes how the different types of federation compete with a typical global pure-play CDN for CPs, so that clarifies the incentive conditions of the different types of federation and compares their competitiveness in terms of revenue increase.

III. SYSTEM MODEL AND PROBLEM FORMULATION

CDN, CP, and consumer. We consider a CDN market that is composed of N geographically separate regions. There are usually a single Telco-CDN in each region (e.g., AT&T in USA or KT in South Korea) providing a local CDN service and one pure-play CDN whose service covers all regions.\(^1\) This implies that a duopoly market is formed by a Telco-CDN and the pure-play CDN to attract local CPs’ subscription in each region (see Fig. 1). We denote the set of the entire regions \(\mathcal{I} = \{1, 2, \ldots, N\}\). Since we have only one CDN for each region \(i\), we also use \(i \in \mathcal{I}\) to index a Telco-CDN of a region, and specially use the special index ‘0’ to denote a global pure-play CDN like Akamai. Each region \(i \in \mathcal{I}\) covers \(\gamma_i\) and \(\omega_i\) portions of end consumers and CPs, those are normalized terms over the total consumer and CP populations of the whole CDN market, respectively, with \(\sum_{i \in \mathcal{I}} \gamma_i = \sum_{i \in \mathcal{I}} \omega_i = 1\).

\(^1\)Our model can also be extended to oligopoly for each region by incorporating their competition.
**Telco-CDN Federation types.** Telco CDNs and the pure-play CDN compete to attract CPs, where we consider the following one benchmark and two types of Telco-CDN federations, depending on the degree of Telco CDNs’ cooperation. Here, we introduce a conceptual difference between the types of federation, and we will revisit them to account for the Telco benchmark and two types of Telco-CDN federations, CDN compete to attract CPs, where we consider the following T3.

**Full federation:**
- Our goal and problem formulation.

**Partial federation:**
- No federation benchmark: In each region, a Telco-CDN individually competes with the pure-play CDN to attract local CPs in the region. T2.
- Partial federation: All Telco-CDNs in different regions form a partial federation agreement to physically share the consumer coverages to increase their competition power against the global pure-play CDN. A Telco-CDN’s CPs can access not only local consumers but those in all the other Telco-CDNs’ region. However, they only maximize their own economic profits by individually deciding pricing strategies under the physical sharing.

**Full federation:**
- Besides physical sharing, there exists a centralizer for making price decision for all Telco-CDNs to compete with the pure-play CDN, in order to maximize all Telco-CDNs’ total profit. To be fair to all, the centralizer then properly decides revenue sharing among Telco-CDNs. Therefore, Telco-CDNs under full federation share the consumer coverages as well as pricing strategies and revenue.

**Our goal and problem formulation.** We aim at understanding how Telco CDNs and the pure-play CDN run their content distribution business under their market competition for each of the three federation cases, and predicting the viability of various federation types. To that end, we formulate a two-stage dynamic game, as summarized in Fig. 1, in what follows:
- **Stage I (Pricing in CDN providers):** Telco-CDNs under a particular federation case and the pure-play CDN choose their regional prices as their federation strategies by predicting the CPs’ responses with respect to the price.
- **Stage II (CPs’ selection of competitive CDN services):** CPs in each region respond by selecting either the local Telco-CDN, the pure-play CDN or neither. Thus, the strategy set of a CP in region \(i\) becomes \(S_i \triangleq \{a, i, n\}\), where we use the ‘a’, ‘i’, and ‘n’ to refer a CP’s selection of the pure-play CDN, the Telco-CDN \(i\), and no CDN subscription, respectively. We denote the vectors \(p = (p_i : i \in I)\) and \(q = (q_i : i \in I)\), where \(p_i\) is the price chosen by the region-i Telco-CDN and \(q_i\) is the price of the region \(i\) charged by the pure-play CDN (see Fig. 1). Note that the pure-play CDN is allowed to charge a different price for a different region, and the choices of \(p\) and \(q\) will depend on how Telco-CDNs federate and how they decide to compete with the pure-play CDN.

To complete the game formulation, it remains to describe the model of CDNs’ utility functions, and the revenue functions of different CPs, which correspond to the payoff functions of CDN players and CP players in our dynamic game-theoretic formulation.

**Utility functions of CPs.** A CP’s utility would be affected by various factors, of which we focus on the CDN service quality, coverage, and service fee. To model this, we consider the following utility function \(u_i : S_i \times \mathbb{R}^N \rightarrow \mathbb{R}\) of a CP in region \(i\):

\[
    u_i(s, p, q) = \begin{cases} 
    \left( \gamma_i \cdot (\theta_i \cdot (\alpha_i - p_i)) \right) & \text{if } s = 'a', \\
    \gamma_i \cdot (\theta_i \cdot (\alpha_i - q_i)) & \text{if } s = 'i', \\
    0 & \text{if } s = 'n',
    \end{cases}
\]

where \( \alpha_i \) and \( \alpha_a \) are the service qualities provided the Telco-CDN \(i\) and the pure-play CDN, and \( \theta_i \), following a uniform random variable over the interval \([0, 1]\), represents the unique willingness to pay of a CP in region \(i\). Following the same uniform distribution independently, different CPs have different \( \theta_i \)’s in general. Thus, \( \theta_i \cdot \alpha_i \) quantifies the revenue per consumer by using CDN service with quality \( \alpha_i \), where \( s \in S_i \). In this paper, we assume that \( \alpha_i > \alpha_a \) for all \( i \in I \), and normalize \( \alpha_a = 1 \), showing that the service quality of Telco-CDNs is better than that of the pure-play CDN due to the advantage of integration of traffic engineering with content distribution. Moreover, we assume that all the consumer population want to access the contents in any CP, thus the utility of CP is proportional to the consumer coverage of CDN provider, \( \gamma_i \) or \( \sum_{i \in I} \gamma_i \).

**CDN Revenue.** The revenue of a CDN provider (a Telco-CDN or the pure-play CDN) depends on the service price, the consumer coverage, the number of subscribing CPs (or market share). The market share of a CDN provider in region \(i\) is determined by the portion of locally-subscribing CPs whose willingness to pay \( (\theta_i)\) is randomly distributed in the range \([0, 1]\). We denote \( \Theta_i \) and \( \Theta_a,i \) as the sets of CPs in \([0, 1]\) in region \(i\) who select Telco-CDN \(i\) and the pure-play CDN, respectively. For a Telco-CDN under non-federation or the partial federation, its revenue does not depend on other Telco-CDNs’ pricing strategies or their revenues. First, the revenue of Telco-CDN \(i\) under non-federation is given by:

\[
    \pi_i(\gamma_i, p_i) = \int_{\Theta_i} \omega_i \cdot \gamma_i \cdot p_i \ d\theta_i, \quad (2)
\]

Moreover, the revenue of Telco-CDN \(i\) under partial federation \( I^p \subseteq I \) is given by:

\[
    \pi_i\left( \sum_{i \in I^p} \gamma_i, p_i \right) = \int_{\Theta_i} \omega_i \sum_{i \in I^p} \gamma_i \cdot p_i \ d\theta_i. \quad (3)
\]

Finally, under full federation, a centralizer sets a price vector \(p\) and a revenue sharing policy needed to distribute the total revenue to all Telco-CDNs. Thus, the revenue of Telco-CDN \(i\) under full federation \( I^F \subseteq I \) is given by:

\[
    \pi_i\left( \sum_{i \in I^F} \gamma_i, p_i \right) = \pi_i(\gamma_i) + \\
    \sigma_i \left\{ \sum_{i \in I^F} \left( \int_{\Theta_i} \omega_i \cdot \left( \sum_{i \in I^F} \gamma_i \cdot p_i \ d\theta_i \right) - \pi_i(\gamma_i) \right) \right\}, \quad (4)
\]

where \( \sigma_i > 0 \) is a revenue portion of Telco-CDN \(i\) in full federation \( I^F \), and \( \sum_{i \in I^F} \sigma_i = 1 \). A vector \( \sigma = (\sigma_i : i \in I) \)
$I^F$) represents a revenue sharing policy among Telco-CDNs under full federation $I^F$. Finally, the revenue of the pure-play CDN is given by:

$$\pi_a\left(\sum_{i \in I} \gamma_i, q_i\right) = \sum_{i \in I} \pi_{a,i}\left(\sum_{i \in I} \gamma_i, q_i\right)$$

$$= \sum_{i \in I} \left(\int_{a_{i,i}} \omega_i \cdot \sum_{i \in I} \gamma_i \cdot q_i \, d\theta_i\right), \quad (5)$$

where the term $\pi_{a,i}(\sum_{i \in I} \gamma_i, q_i)$ denotes the revenue from region $i$, thus the total revenue of the pure-play CDN is defined by their sum.

### IV. Competitive CDN-Market Analysis: Telco-CDNs vs. Pure-Play CDN

We first analyze a competitive CDN market, which includes $N$ duopoly regions, each played between a local Telco-CDN and the pure-play CDN. By analyzing the two-stage dynamic game in Fig. 1, we want to understand i) the CPs’ equilibrium choices in each region between the local Telco-CDN or the pure-play CDN, and ii) the Telco-CDNs’ pricing decisions against the pure-play CDN. In this section, we consider the traditional non-federation case where each Telco-CDN competes with the pure-play CDN but the results can be extended to partial and full federations in Section V.

We analyze a long-term competition in the context of the stable tension between the two key factors, which are the quality of service ($\alpha_i, \alpha_a$) and the consumer density (or coverage), ($\gamma_i$). According to the tension of them, the market analysis can be divided into two regimes, one is the coverage-dominant regime where a Telco-CDN’s advantage (quality of service) is lower than ones disadvantage (consumer coverage), i.e., $\alpha_i \leq 1/\gamma_i$. Similarly, the other is the quality-dominant regime, i.e., $\alpha_i > 1/\gamma_i$. In each regime, one of CDN providers dominates the other in terms of market share of CPs.

By the backward induction, we can find the equilibrium of the two-stage dynamic game. In each region, for given pricing policies of Telco-CDNs, in Stage II a CP with preference $\theta_i$ compares its utilities in (1) by choosing either the Telco-CDN, the pure-play CDN or neither, to pick the one giving it the highest utility. Then, we have the following result.

**Proposition IV.1 (CPs’ equilibrium selection in Stage II)**

In each region $i \in I$, CPs’ partition between Telco-CDN i’s and the pure-play CDN’s services depends on whether the Telco-CDN’s quality improvement dominates the pure-CDN’s global coverage or not:

- **Quality-dominant regime ($\alpha_i > 1/\gamma_i$):** The Telco-CDN attracts the CPs with higher preference $\theta_i \in \left[\frac{\alpha_i \gamma_i - \gamma}{\alpha_i \gamma_i - 1}, 1\right]$ and only CPs with lower preference $\theta_i \in \left[\gamma, \frac{\alpha_i \gamma_i - \gamma}{\alpha_i \gamma_i - 1}\right]$ choose the pure-play CDN, the rest CPs choose neither CDN providers.

- **Coverage-dominant regime ($\alpha_i \leq 1/\gamma_i$):** The pure-play CDN attracts the CPs with higher preference $\theta_i \in \left[\frac{\gamma - \gamma \alpha_i}{1 - \alpha_i \gamma_i}, 1\right]$ and only CPs with lower preference $\theta_i \in \left[\frac{\gamma - \gamma \alpha_i}{1 - \alpha_i \gamma_i}, \frac{\gamma - \gamma \alpha_i}{\alpha_i \gamma_i - 1}\right]$ choose the Telco-CDN, the rest CPs choose neither CDN provider.

**Proof:** For an arbitrary region $i$, and the utilities of CP (1) Telco-CDN and the pure-play CDN are given by $u_i = \gamma_i(\theta_i \alpha_i - p_i)$ and $u_a = \theta_i - q_i$, respectively. In a quality-dominant regime, since $\alpha_i > 1/\gamma_i$, it is satisfied that $u_i > u_a$ if the type of CPs is in $\theta_i \in \left[\frac{\alpha_i \gamma_i - \gamma}{\alpha_i \gamma_i - 1}, 1\right]$. Also, the CPs in $\theta_i \in [\gamma, \frac{\alpha_i \gamma_i - \gamma}{\alpha_i \gamma_i - 1}]$ satisfy $u_a > u_i$. Similarly with the quality-dominant regime, we can derive the CPs’ equilibrium selection in a coverage-dominant regime.

A CDN company with superior service (in term of quality or coverage) can attract high-end CPs with large $\theta$ values who are willing to pay more. This also indicates the federation advantage to improve Telco-CDNs’ competitiveness by increasing coverage. Note that, the results can be extended to partial and full federations, by replacing the individual consumer coverage, $\gamma_i$, with the federation’s consumer coverage, $\sum_{i \in I} \gamma_i$.

In Stage II of the two-stage dynamic game, a Telco-CDN and the pure-play CDN predict the equilibrium behaviors and the market shares of CPs according to their decisions in Stage I, thus they also play with the best responses with respect to a given price of competitor. By simultaneously analyzing how each Telco-CDN and pure-play CDN compete with each other to converge to equilibrium and stable prices, we find the equilibrium prices in which both providers do not have incentives to deviate. The equilibrium prices are formed as Theorem IV.1.

**Theorem IV.1 (CDNs’ equilibrium prices and revenues)**

The equilibrium prices of a Telco-CDN and the pure-play CDN in region $i \in I$ are as follows:

- **Quality-dominant regime ($\alpha_i > 1/\gamma_i$):** The high quality service enables Telco-CDN $i$ to charge a higher price $p_i^*$ than the pure-play CDN $q_i^*$, and the two prices are given by:

$$p_i^* = \frac{2\alpha_i(\alpha_i \gamma_i - 1)}{4\alpha_i - \gamma_i - 1}, \quad q_i^* = \frac{\alpha_i \gamma_i - 1}{4\alpha_i - \gamma_i - 1}.$$  

Then, the revenues of two CDN providers in region $i$ are given by:

$$\pi_i^* = \frac{4\alpha_i \gamma_i^2 \omega_i (\alpha_i \gamma_i - 1)}{(4\alpha_i - \gamma_i - 1)^2}, \quad \pi_a^* = \frac{\alpha_i \gamma_i \omega_i (\alpha_i \gamma_i - 1)}{(4\alpha_i - \gamma_i - 1)^2}. \quad (6)$$

- **Coverage-dominant regime ($\alpha_i \leq 1/\gamma_i$):** The high coverage service enables the pure-play CDN to charge a higher price $q_i^*$ in region $i$, than the Telco-CDN $i$, and the two prices are given by:

$$p_i^* = \frac{\alpha_i (1 - \alpha_i \gamma_i)}{4 - \alpha_i \gamma_i}, \quad q_i^* = \frac{2(1 - \alpha_i \gamma_i)}{4 - \alpha_i \gamma_i}.$$  

Then, the revenues of two CDN providers in region $i$ are given by:

$$\pi_i^* = \frac{\alpha_i \gamma_i \omega_i (1 - \alpha_i \gamma_i)}{(4 - \alpha_i \gamma_i)^2}, \quad \pi_a^* = \frac{4\omega_i (1 - \alpha_i \gamma_i)}{(4 - \alpha_i \gamma_i)^2}. \quad (7)$$

Due to space limitation, we present the proof in [18]. As the CDN company (e.g., the Telco-CDN $i$ with dominant service quality or with dominant service coverage) can cover high-end CPs with high willingness to pay, it can also charge
a higher price as compared to its competitor in the same region. Therefore, we see that the dominant CDN company’s price is two times higher than the competitor’s. Therefore, in region $i$, Telco-CDN $i$’s revenue is much higher than the pure-play CDN’s in the quality-dominant regime, and the pure-play CDN’s revenue is also much higher than the Telco-CDN $i$’s revenue in the coverage-dominant regime.

When $\gamma_i = 1/\alpha_i$, there is no difference between the market powers of the CDN providers, thus CPs have the same utility regardless of the subscription type of CDN providers. Therefore, each CDN provider should decrease its price to attract the CPs, leading to zero prices and zero revenues.

**Corollary IV.1 (Perfect competition)** In each region $i \in \mathcal{I}$, we call the point satisfying $\gamma_i = 1/\alpha_i$ as the perfect competition point in which the equilibrium prices of Telco-CDN $i$ and the pure-play CDN are zeros, thus the revenues of both of CDN providers are zero.

The proof is shown in [18]. We numerically illustrate the revenues of Telco-CDN and pure-play CDN in region $i \in \mathcal{I}$ in Fig. 2. We assume that $\alpha_i = 1.5$, thus the quality-dominant regime is when $\gamma_i > 2/3$ where $1/\alpha_i = 1/1.5 = 2/3$ and the coverage-dominant regime is when $\gamma_i \leq 2/3$. In each regime, the CDN company with dominant service quality or service coverage earns much more revenue than the competitor. Moreover, at the perfect competition point, both of Telco-CDN $i$ and the pure-play CDN cannot make revenues due to the severe competition.

Now, we focus on the Telco-CDN’s revenue. There are two ways to dominate the CDN market, which are i) increasing the quality of service ii) increasing the consumer service coverage. However, in reality, it is hard to increase the quality of service since it might be a high-cost and time-consuming operation. Therefore, another and more plausible way to increase revenue is to enlarge service coverage, by federating with other Telco-CDNs.

**V. Federation of Telco-CDNs**

In this section, we study whether and how Telco-CDNs should federate to compete against the pure-play CDN. The key questions are i) whether the federation with the goal of increasing consumer coverage always increases the revenues of Telco-CDNs and ii) if not, how we should design a federation policy to include all Telco-CDNs. As discussed in Section III, Telco-CDNs can choose to federate "physically” and/or further “economically.”

**T1. No federation:** The Telco-CDN in each region individually competes with the pure-play CDN and selects the price that maximizes its own revenue, i.e.,

$$p_i^* = \arg \max_{p_i} \pi_i(\gamma_i, p_i),$$

where $\pi_i(\gamma_i, p_i)$ is given in (2), and the maximized revenue is $\pi_i(\gamma_i, p_i^*)$ or simply $\pi_i(\gamma_i)$.

**T2. Partial federation:** All Telco-CDNs in a partial federation $\mathcal{I}^p \subseteq \mathcal{I}$ share their consumer coverages physically but do not cooperate economically to decide prices or share revenue. Thus each Telco-CDN still decides the price to maximize its own revenue individually, given that such physical federation increases its revenue, i.e.,

$$p_i^* = \arg \max_{p_i} \pi_i\left(\sum_{i \in \mathcal{I}^p} \gamma_i, p_i\right),$$

s.t. $\pi_i\left(\sum_{i \in \mathcal{I}^p} \gamma_i\right) > \pi_i(\gamma_i), \forall i \in \mathcal{I}^p$,

where $\pi_i(\sum_{i \in \mathcal{I}^p} \gamma_i, p_i)$ is given in (3). The maximized revenue is $\pi_i(\sum_{i \in \mathcal{I}^p} \gamma_i, p_i^*)$ or simply $\pi_i(\sum_{i \in \mathcal{I}^p} \gamma_i)$.

**T3. Full federation:** All Telco-CDNs in a full federation $\mathcal{I}^F \subseteq \mathcal{I}$ cooperate not only physically but also economically. To that end, a centralizer decides a price vector $p = (p_i : i \in \mathcal{I}^F)$ that first maximizes the total federation’s revenue. Then, it distributes the revenue to all Telco-CDNs based on a proper revenue sharing policy, given that such full federation increases the total revenue, i.e.,

$$p^* = \arg \max_{p} \sum_{i \in \mathcal{I}^F} \pi_i\left(\sum_{i \in \mathcal{I}^F} \gamma_i, p_i\right),$$

s.t. $\sum_{i \in \mathcal{I}^F} \pi_i\left(\sum_{i \in \mathcal{I}^F} \gamma_i\right) > \sum_{i \in \mathcal{I}^F} \pi_i(\gamma_i)$,

where $\pi_i(\sum_{i \in \mathcal{I}^F} \gamma_i, p_i)$ is given in (4). The maximized revenue is $\pi_i(\sum_{i \in \mathcal{I}^F} \gamma_i, p_i^*)$ or simply $\pi_i(\sum_{i \in \mathcal{I}^F} \gamma_i)$.

As mentioned in Section IV, Proposition IV.1 can be readily extended to analyze CPs’ equilibrium under partial and full federations by replacing the individual consumer coverage with the federation’s consumer coverage. Fig. 3 shows how a Telco-CDN’s revenue $\pi_i(\gamma)$ changes with its consumer coverage $\gamma$ when competing with the pure-play CDN, then we observe that:

- **Regime I:** It first increases with $\gamma$ till $4/\alpha$ due to the increase of consumer coverage to compete with the pure-play CDN in the coverage-dominant regime.
- **Regime II:** It decreases to zero at the perfect competition when $\gamma = 1/\alpha$.
- **Regime III:** It finally increases again in the quality-dominant regime as its coverage gap reduces. At the point $\hat{\gamma} \approx 1.05/\alpha$, the revenue is the same as the maximum value in the coverage-dominant regime, i.e., $\pi_i(\frac{1}{\alpha}) = \pi_i(\hat{\gamma}) = \frac{1}{18}$.
We assume that $\gamma_2 \geq \gamma_1$ without loss of generality and denote $\gamma_+$ by the sum of consumer coverages, i.e., $\gamma_+ = \gamma_1 + \gamma_2$. Moreover, different $\gamma_+$ values can lead to the same revenue level. For example, when $\gamma_+ \in [\frac{\gamma_2}{\alpha}, 1/\alpha]$, there are two other coverage values, $[\pi^{-1}_i(\gamma_+)]^I$ in regime I and $[\pi^{-1}_i(\gamma_+)]^II$ in regime II achieving the same revenue level as $\gamma_+$ in region III. Similarly, when $\gamma_+ \in [1/\alpha, 1]$, there are also two other coverage values $[\pi^{-1}_i(\gamma_+)]^I$ in region I and $[\pi^{-1}_i(\gamma_+)]^II$ in regime II. Throughout this section, we focus on two Telco-CDNs’ federation (Telco-CDNs 1 and 2) to clearly deliver the engineering insights, and we can extend to multiple Telco-CDNs’ federation without major change of insights (see the technical report [18]).

A. Partial Federation

We first study the conditions of the incentives for Telco-CDNs to form a partial federation. Both Telco-CDNs 1 and 2 have incentives to form a partial federation if their revenues satisfy the incentive constraint (9), i.e., $\pi_1(\gamma_1 + \gamma_2) > \pi_1(\gamma_1)$ and $\pi_2(\gamma_1 + \gamma_2) > \pi_2(\gamma_2)$.

**Theorem V.1 (Incentive conditions for partial federation)**

Both Telco-CDNs 1 and 2 have incentives to form a partial federation if one of the following conditions about federated coverage $\gamma_+$ are satisfied:

1. $0 < \gamma_+ \leq \frac{1}{\alpha}$ or $\gamma_+ < \gamma_+ \leq 1$.
2. $\frac{\gamma_+}{\alpha} < \gamma_+ \leq 1/\alpha$ and $\gamma_2 < [\pi^{-1}_2(\gamma_+)]^I$.
3. $1/\alpha < \gamma_+ \leq \gamma_+$ and $\gamma_2 < [\pi^{-1}_2(\gamma_+)]^I$ (or $\gamma_2 > [\pi^{-1}_2(\gamma_+)]^II$).

Due to space limitation, we present the proof in [18]. In C1, all Telco-CDNs in the partial federation have incentives to federate if the total consumer coverage is small enough (smaller than $\frac{1}{\alpha}$) or large enough (larger than $\gamma_+$) to cause perfect competition with the pure-play CDN. The incentive intervals of the total consumer in C1 are safety on the revenue increasing interval (regime I and III) in Fig. 3.

In C2, since the revenue function is decreasing as shown in Fig. 3, Telco-CDNs may not have incentive to federate. For example, if we assume $\gamma_2 = \frac{1}{\alpha}$, then the revenue of Telco-CDN 2 decreases due to mildly adding $\gamma_1$ to $\gamma_+ < 1/\alpha$, i.e., $\pi_2(\gamma_1) < \pi_2(\frac{\gamma_1}{\alpha})$. Furthermore, if $\gamma_1 \leq \gamma_2 \leq [\pi^{-1}_2(\gamma_+)]^I$, we can ensure each Telco-CDN’s revenue does not reduce after federation.

In C3, although the revenue function is increasing in this interval as shown in Fig. 3, there may exist Telco-CDNs with no incentive to federate, since there is a higher revenue interval (e.g., $[\pi^{-1}_2(\gamma_+)]^I$, $[\pi^{-1}_2(\gamma_+)]^II$ in Fig. 3) than the revenue of total consumer coverage in this interval (e.g., $[1/\alpha, [\pi^{-1}_2(\gamma_+)]^III$ in Fig. 3). Therefore, the partial federation in this interval provides enough incentives to both Telco-CDNs either when (i) both consumer coverages are small (less than $[\pi^{-1}_2(\gamma_+)]^I$), or (ii) one of them is small (less than $[\pi^{-1}_2(\gamma_+)]^II$) and the other is large (larger than $[\pi^{-1}_2(\gamma_+)]^III$).

B. Full Federation and Revenue-Sharing

We now study the case of full federation, where our major interests are: (i) finding the incentive conditions for the full federation and (ii) investigating the impact of the revenue-sharing policies to distribute the total revenue of the federation to Telco-CDNs. We first present Proposition V.1, which compares the total revenues for different types of federation.

**Proposition V.1 (Total revenue of federation)**

For any subset of Telco-CDNs $\mathcal{I} \subseteq \mathcal{I}$, the total revenue under full federation of $\mathcal{I}$ is the same as the total revenue under the partial federation of $\mathcal{I}'$.

The proof is shown in [18]. The above proposition implies that the federation type, whether it is partial or full, does not affect the total federation’s revenue. This is interesting, because the pricing decision mechanisms of partial and full federations highly differ, as explained in (9) and (10). This result is caused by the feature of Telco-CDNs’ federation, being independent of market shares among different Telco-CDNs, i.e., Telco-CDN $i$ is unable to attract the subscribers from different regions’ CPs.

However, we highlight that the federation type is still important on the individual revenue of each Telco-CDN, because the same total revenue does not guide how much each individual Telco-CDN should be distributed its own revenue in the full federation. The individual revenue of a Telco-CDN under the full federation is defined by (4), thus the full federation has incentive to federate if the incentive constraint in (10) holds. Theorem V.2 describes the incentive conditions of the full federation of Telco-CDNs 1 and 2.

Prior to describing Theorem V.2, we respectively define two constant parameters in regimes II and III:

$$\gamma_{II} = \frac{24 - 4\sqrt{22}}{5} \in (\frac{1}{\gamma_+}, 1/\alpha)$$

and

$$\gamma_{III} = (1/\alpha, \gamma_+)$$

is the unique solution to the equation:

$$32\alpha^4 \gamma_{III} - 208\alpha^3 \gamma_{III}^2 - 560\alpha^2 \gamma_{III}^3 - 239\alpha \gamma_{III}^4 - 1 = 0.$$  

We describe the derivations and analytical implications of $\gamma_{II}$ and $\gamma_{III}$ in our technical report [18].

**Theorem V.2 (Incentive conditions for full federation)**

Both Telco-CDNs 1 and 2 have incentives to form the full federation if one of the following conditions about federated coverage $\gamma_+$ are satisfied:

We use superscript I, II, and III to emphasize that $[\pi^{-1}_i(\gamma_+)]^I$, $[\pi^{-1}_i(\gamma_+)]^II$ and $[\pi^{-1}_i(\gamma_+)]^III$ are in regime I, regime II and regime III in Fig. 3, respectively.
C1. $0 < \gamma_+ \leq \gamma^{II}$ or $\gamma^{III} < \gamma_+ \leq 1$.

C2. $\gamma^{II} < \gamma_+ < 1/\alpha$ and $\gamma_2 > \gamma_+ - \epsilon_1(\gamma_+)$,

C3. $1/\alpha < \gamma_+ \leq \gamma^{III}$ and $\gamma_2 > \frac{1}{\alpha} - \epsilon_2(\gamma_+)$.

Note that $\epsilon_1$ and $\epsilon_2$ are the functions of $\gamma_+$ and [18] provides the analytic values of $\epsilon_1$ and $\epsilon_2$ as well as the proof of Theorem V.2. Intuitively, the incentive condition of full federation is looser than that of partial federation, since by revenue sharing, a highly profitable Telco-CDN is able to economically compensate for the revenue loss of the other Telco-CDN. Therefore, if the total revenue of full federation exceeds the total revenue of non-federation, they have incentive to federate under some revenue sharing policy that encourages the federation.

In C1, Telco-CDNs have incentive to federate irrespective of $\gamma_1$ and $\gamma_2$. In particular, for $\frac{1}{\alpha} < \gamma_+ < \gamma^{II}$ and $\gamma^{III} < \gamma_+ \leq \gamma$, Telco-CDNs always have incentives under the full federation, but not always under the partial federation. Under the full federation, the revenue loss of Telco-CDN 1 (resp. 2) is always compensated by the revenue gain of Telco-CDN 2 (resp. 1).

Moreover, in C2, there is no incentive to partially federate in this interval due to the threat of perfect competition. However, we can find a pair of $\gamma_1$ and $\gamma_2$ providing incentive when $\gamma_1$ is very close to zero. This implies that even though a Telco-CDN is on the decreasing revenue interval, such as on $(\frac{1}{\alpha}, 1/\alpha)$, it can have incentive to federate with a Telco-CDN with very small consumer coverage.

Finally, in C3, when $1/\alpha < \gamma_+ < \gamma^{III}$, which is similar to the partial federation, a small coverage Telco-CDN and a large coverage Telco-CDN have incentive to federate, but the conditions of Telco-CDNs’ consumer coverage are relaxed by the amount of $\epsilon_2$ due to revenue compensation.

Impact of revenue sharing policies. Now, we study the impact of the revenue sharing policies to reach a win-win situation. From the definition (4), we consider two revenue sharing policies that distribute the total revenue increase from federation of Telco-CDNs: equal-sharing and proportional-sharing policies. For the full federation $\mathcal{I}^F \subseteq \mathcal{I}$, in equal-sharing, as the name implies, the revenue is shared equally, i.e., $\sigma^E = (1/|\mathcal{I}^F|, 1/|\mathcal{I}^F|, \ldots, 1/|\mathcal{I}^F|)$. In proportional-sharing policy, the revenue is distributed in proportion to the consumer coverage, i.e., $\sigma^P = (\frac{\gamma_1}{\sum_{i \in \mathcal{I}^F} \gamma_i}, \frac{\gamma_2}{\sum_{i \in \mathcal{I}^F} \gamma_i}, \ldots, \frac{\gamma_{|\mathcal{I}^F|}}{\sum_{i \in \mathcal{I}^F} \gamma_i})$. We numerically compare these two revenue sharing policies in the following section.

VI. Numerical Results

In this section, we provide numerical results to further illustrate the analytical results in Section V and summarize the implications of the Telco-CDNs’ federation. Under full federation, we further investigate the impacts of different revenue sharing policies on Telco-CDNs under the full federation.

A. Incentive Comparison between Partial and Full Federations

In Section V, we studied the incentive conditions for partial and full federation as described in Theorems V.1 and V.2. However, it is difficult to analytically compare them, thus now we compare them numerically by fixing $\alpha_1 = \alpha_2 = 1.25$ and focus on the area of $\gamma_1 + \gamma_2 \leq 0.8$ as shown in Fig. 4.

When the federation is in the perfect competition with the pure-play CDN if $1/(\gamma_1 + \gamma_2)$ equals both Telco-CDNs’ qualities of service, i.e., $\alpha_1 = \alpha_2 = 1/(\gamma_1 + \gamma_2)$. Here, $\gamma_1 + \gamma_2 = 1/1.25 = 0.8$ is the perfect competition point. Therefore, the aggregate coverage $\gamma_1 + \gamma_2$ equals or approaches to 0.8, neither partial or full federation exists as they will reach the non-profitable perfect competition outcome with the pure-play CDN. Finally, when $\gamma_1$ and $\gamma_2$ are asymmetrically distributed (e.g., $(\gamma_1, \gamma_2) = (0.1, 0.5)$ or $(0.5, 0.1)$ in Fig. 4), partial federation does not exist, as the large-coverage Telco-CDN does not increase coverage greatly but suffers from the pure-play CDN’s competition. But under full federation, small-coverage Telco-CDN will share its revenue increase to compensate the revenue loss of the large-coverage Telco-CDN, thus both of them can have incentive to fully federate.

B. Impact of Equal and Proportional Revenue Sharing

We now numerically study the revenues of a particular Telco-CDN (e.g., Telco-CDN 1) for two different federation types, partial and full. In particular, under full federation, we consider two revenue sharing policies: (i) equal-sharing and (ii) proportional-sharing in distributing the total revenue increase due to federation to the federation participants. From the definition in (4) of Telco-CDN’s revenue under full federation, we apply the equal-sharing policy with $\sigma^E = (1/2, 1/2)$ and the proportional-sharing policy with $\sigma^P = (\gamma_1/\gamma_1 + \gamma_2, \gamma_2/\gamma_1 + \gamma_2)$. Without loss of generality, the consumer coverage of Telco-CDN 2 is given by $\gamma_2 = 0.2$, and that of Telco-CDN 1 is varied from 0 to 0.8. We further assume that $\alpha_1 = \alpha_2 = 1.5$, i.e., homogeneous service quality for both Telco-CDNs.

In Figs. 5(a) and 5(b), we show the numerical results in the quality-dominant regime thus both of Telco-CDNs
have incentives to federate under all types of federation. Under non-federation, Telco-CDN 1 has zero revenue when \( \gamma_1 = 1/\alpha_1 = 1/1.5 \approx 0.67 \) due to the perfect competition, and Telco-CDN 2 has fixed revenue due to the given coverage, \( \gamma_2 = 0.2 \). Under the partial federation, both of Telco-CDNs 1 and 2 have the same amount of revenues, since they have the same consumer coverage, \( \gamma_1 + \gamma_2 \). However, in the full federation, the revenues of Telco-CDNs are highly affected by the revenue sharing policy: With the equal-sharing, Telco-CDN 1 has less revenue than Telco-CDN 2 before the perfect competition, since Telco-CDN 1’s revenue in non-federation is less than the Telco-CDN 2’s, whereas with the proportional sharing, due to \( \gamma_2 = 0.2 < \gamma_1 \), Telco-CDN 1’s revenue exceeds that of Telco-CDN 2.

VII. CONCLUSION AND FUTURE WORK

In this paper, we analyzed the competitive CDN market among new Telco-CDNs and a global pure-play CDN. We take a two-stage dynamic game model that characterizes how multiple Telco-CDNs compete with the pure-play CDN for CPs in terms of two different federation types (partial and full federations) and revenue sharing policies (for the full federation). Our analysis reveals that (i) the federation type differently affects the competitiveness of Telco-CDNs in terms of revenue, but both of partial and full federations may not help to avoid the perfect competition with the pure-play CDN. In particular, under the full federation, how to distribute the total revenue to each individual Telco-CDNs is crucial. Our work has several possible ways to extend. First, we can consider the case of multiple Telco-CDs and study whether and how they federate. In our technical report [18], we present the results for for homogeneous Telco-CDNs and will further consider heterogeneous Telco-CDNs in the future. Second, it is interesting to analytically study the condition of federation stability, where cooperative game theory would become a useful tool.

REFERENCES