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Capacity Allocation and Pricing Strategies for New Wireless Services

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Indoor cell phone users often suffer poor connectivity. One promising solution to this issue, femtocell technology, has been rapidly developed and deployed over the past few years. One of the biggest challenges facing femtocell deployment is the lack of a clear business model. This study investigates the economic incentive for cellular operators (also called macrocell operators) to enable femtocell service by leasing spectrum resources to independent femtocell operators. We model the interactions between a macrocell operator, a femtocell operator, and end-users as a three-stage dynamic game, and derive the equilibrium pricing and capacity allocation decisions. We show that when spectrum resources are very limited, the macrocell operator has more incentive to lease spectrum to the femtocell operator, as femtocell services can help cover more users and improve the utilization efficiency of the limited spectrum resource. However, when the total spectrum resource is large, femtocell service offers significant competition to macrocell service and, as a result, the macrocell operator has less incentive to enable femtocell service. We also show the impact of the additional operational costs and limited coverage of femtocell service on equilibrium decisions, consumer surplus, and social welfare.

Key words: wireless service; capacity allocation; pricing; game theory
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1. Introduction

Today, there are over 4.5 billion mobile phone users in the world (eMarketer 2014), and many of them experience poor indoor reception at home or the office. This is because, in the current cellular network (also called the *macrocell* network), a base station covers an area of a radius from several hundred meters to several kilometers. High-frequency and low-power wireless signals, however, often have difficulty in effectively traveling from an outdoor macrocell base station to indoor cell phones through walls. As a result, indoor cell phone users often experience dropped calls and reduced wireless data rates (Sandler 2009).

As one promising solution to the indoor reception problem, *femtocell technology*, has been rapidly developed and deployed over the past few years. Figure 1 provides an illustration of four homes covered by one macrocell base station, three of which have installed femtocell base stations. Femtocell is a small base station with a size similar to a wireless router. An indoor femtocell base station is much closer to users' indoor cell phones, hence, there is little channel attenuation or fading. It can pick up the cell phones' signals much more effectively, and then deliver voice and data traffic to the cellular network through users' home wireline Internet connection. Femtocell technology can significantly increase the quality of voice calls and improve the speed of data communications (Shetty et al. 2009).

Major operators worldwide are enthusiastic about femtocell technology due to its capability of improving customers' experiences. In the United States, AT&T, Sprint Nextel, and Verizon Wireless are already offering femtocell services to their customers. T-Mobile and Vodafone in Europe, NTT DoCoMo and Softbank in Japan, and Unicom in China have been testing the technology and planning to implement nationwide femtocell services. The UK research firm Informa Telecoms & Media reported that femtocell deployments had more than doubled from 2009 to 2010, with substantially increasing numbers of tier-one operators joining this trend. Global femtocell

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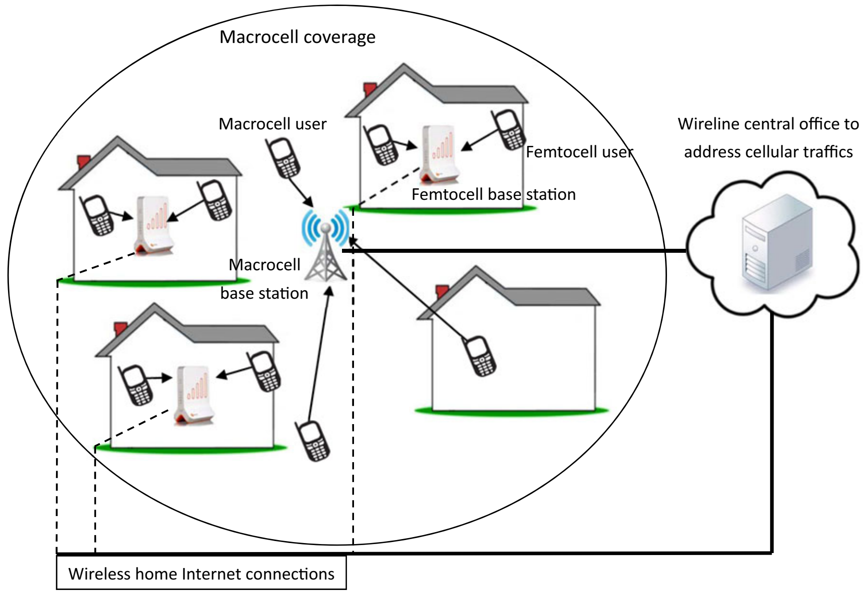
Figure 1 Coexistence of Femtocell Service and Macrocell Service, in Which a Macrocell and Three out of Four Femtocells are Deployed



deployments is expected to increase from 2.1 million units in 2011 to 87.3 million units in 2016 (Informa, Telecoms & Media 2011, 2012).

However, one of the biggest challenges to an operator's large-scale femtocell deployment is the lack of a clear business model. As Emin Gurdenli, Chief Technology Officer of Deutsche Telekom AG's T-Mobile United Kingdom, put it (The Wall Street Journal 2009): *"The rationale for femtocells is well-established, but a quantitative business case with a clear business model in terms of how we go to market is not there yet."*

The purpose of this study is to develop a quantitative model to examine the business trade-off of femtocell deployments. In particular, we examine a commonly used distributed approach to deploying femtocell service, in which a macrocell operator may lease some of its spectrum resources to a femtocell operator. The femtocell operator serves as a virtual operator, and determines the service provision and pricing independently. There are many examples of distributed deployment of femtocell service in the wireless industry. For example, Sprint leases spectrum to Virgin Mobile USA to provide femtocell service (Gabriel 2009), and BT Mobile uses Vodafone's resource to provide femtocell service (Chambers 2010).

We investigate the following research questions:

- *Should a macrocell operator support femtocell service? How would the operator allocate bandwidth (capacity) resources and make pricing decisions?* The key trade-off here is between additional revenue source and increased market competition: a macrocell operator may increase its revenue by leasing resources to the femtocell operator, meanwhile it will have fewer resources for its own services and face increased market competition from the femtocell operator.
- *How would users choose between femtocell and macrocell services?* By using the indoor femtocell base station, users can avoid the poor reception problem and achieve better quality of service. In contrast, when users choose to use the outdoor macrocell base stations, the service quality highly depends on the user locations and the communication environments (which can be described by a user-dependent spectrum efficiency parameter). As the prices of femtocell and macrocell services may be very different, a user needs to evaluate the trade-off between cost and quality of service.

Different from the studies of femtocell technology in the wireless telecommunications literature, which focus on the technical operations of femtocell services, our study focuses on the business analysis of capacity and pricing decisions as well as the economic trade-off between cooperation and competition. Furthermore, compared to the dual channels studies in the operations management and marketing science literature which generally assumes unlimited supply, we incorporate the constraint of limited supply of spectrum resources. The capacity constraint better reflects the reality of the wireless market – wireless spectrum has become a very scarce resource, because wireless demand has grown dramatically during recent years,

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but the spectrum resources remain limited due to various physical constraints. The capacity constraint, as we will show, significantly affects the operator's decisions (and complicates our analysis). Finally, we model the heterogeneity and the utility functions of users based on the unique characteristics of wireless communications, which makes our model very different from the typical setup in the dual channel literature.

Our main results are summarized as follows:

- *Characterization of equilibrium decisions:* We derive a threshold structure in terms of the spectrum efficiency parameter, which separates users who prefer femtocell or macrocell service. We further characterize the femtocell operator's femtocell price and the macrocell operator's capacity allocation and pricing decisions at equilibrium.
- *Sensitivity analysis of macrocell's total capacity:* We analyze how the spectrum capacity constraint affects the equilibrium decisions. One key (and perhaps counter-intuitive) finding is that the macrocell operator has more incentive to lease spectrum to the femtocell operator when its spectrum capacity is small.
- *Examination of consumer surplus and social welfare:* We show that, in general, femtocell service can increase both the *total* consumer surplus and social welfare. However, some users who originally receive good service in macrocell might experience a payoff drop after the femtocell service is introduced, due to fewer resources being allocated to the macrocell service and a higher macrocell price.

In addition, we examine two extensions of the basic model, with additional femtocell operational cost and with limited femtocell coverage. The rest of the study is organized as follows. Section 3 introduces the network model of macrocell service, which serves as a benchmark for later analysis. Section 4 presents the network model of dual services and analyzes how the operators make optimal capacity allocation and pricing decisions. Sections 5 and 6 extend the results in section 4 by examining the various effects of additional femtocell operational cost and limited femtocell coverage. Section 7 concludes our study and discusses some future research directions.

2. Literature Review

Our work is closely related to two main streams of literature: (i) studies of femtocell deployment in the telecommunication literature; and (ii) studies of dual channel competition in the operations management and marketing science literature.

Most existing work on femtocell deployment in the telecommunication literature (e.g., Chandrasekhar and Andrews 2009) focuses on various technical issues in service provisions, such as access control, resource management, and interference management. Only a few papers discuss the economic issues of femtocells (e.g., Claussen et al. 2007, Yun et al. 2011, Shetty et al. 2009, Chen et al. 2012, Duan et al. 2013), examining the impact of network deployment costs and femtocells' openness to macrocell users. The key difference between our study and the existing literature is that we study the provision of dual services in terms of both spectrum allocations and pricing decisions. Our focus is on the business analysis of the trade-off between cooperation and competition.

Our work is also related to studies on *dual channel management* in the operations management and marketing science literature. Tsay and Agrawal (2004a) provide a comprehensive review of earlier studies on the competition and coordination of dual channel distribution systems. The authors grouped the related studies into two categories based on whether the manufacturer owns the direct channel or the retail store. Our study would belong to the former category. Representative papers in this category are Chiang et al. (2003) and Tsay and Agrawal (2004b). Chiang et al. (2003) consider whether a manufacturer should sell through a direct channel, an exclusive retailer, or a hybrid of both. The key finding is that, at equilibrium, the manufacturer will price in the direct channel, such that all customers will purchase from the retail channel – the direct channel exists not to sell the product, but to influence the independent retailer's price to reduce double marginalization. Tsay and Agrawal (2004b) consider a similar problem setting as in Chiang et al. (2003); however, they assume that the manufacturer and the retailer can control not only the prices, but also the sales efforts. They further exploit several means whereby the manufacturer can mitigate channel conflict between the direct channel and the retail channel, including adjustments of wholesale price, paying a commission to a retailer, and entirely conceding demand fulfillment on the part of the retailer.

The past few years continued to see a good amount of interest in the dual-channel system. Chen et al. (2008) study a setting in which a manufacturer operates both a direct online channel and an independent retail channel. The two channels compete in service, measured as delivery time for the online channel and stocking rate for the retail channel. The prices for the two channels are assumed to be exogenous and the same. The authors incorporate a detailed consumer channel choice model and identify optimal dual channel strategies, that is, when the manufacturer should establish a direct channel,

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a retail channel, or both. Huang and Swaminathan (2009) propose a stylized deterministic demand model, in which each channel relies on prices, degree of substitution across channels, and the overall market potential. Dumrongsiri et al. (2008) investigate the influence of demand variability on prices and manufacturer's incentive to open the direct channel. David and Adida (2014) study competition and coordination in a supply chain, in which a supplier operates a direct channel and also sells through multiple retailers. The direct channel and the retail channels engage in a quantity competition, with the market prices determined by the total order quantities. They show that the supplier benefits from having more retailers in the market and that linear quantity discount contracts can mitigate supply chain inefficiency.

In the context of wireless service deployment, our paper studies the decision of a macrocell wireless service provider (manufacturer) to sell through a direct channel, an independent femtocell operator (retailer), or both.

However, our study has several key differences from the existing dual channel literature. First, we explicitly incorporate the limited capacity constraint into our model, while all of the prior studies discussed above have assumed unlimited capacity. Second, we model the heterogeneity of users based on the unique characteristics of wireless communications. In particular, users have different channel conditions (and thus different evaluations of the same resource allocation) under the macrocell service (direct channel), but have the same maximum channel condition under the femtocell service (retailer channel). In contrast, prior literature on dual channel either assumes that users are homogenous, or users are different in terms of their willingness to pay or sensitivities to delivery time, and such heterogeneity is often independent of the channel choice. Moreover, the users' utility function in our study is also motivated by today's wireless communication technologies, which renders much of the prior simplified models (e.g., linear form of user-specific coefficients) and generic analysis inapplicable. Finally, we characterize the impact of limited femtocell coverage on the new service provision. To the best of our knowledge, none of the prior dual-channel studies have considered such a constraint.

3. Benchmark Scenario: Macrocell Service Only

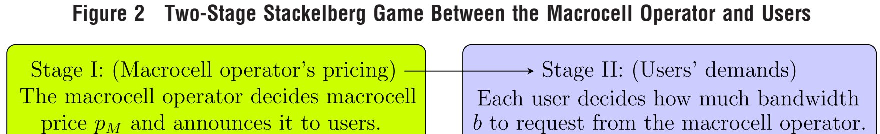
In this study, we consider the interaction between one macrocell operator and one femtocell operator. This is a simplified assumption, as in many countries there often exists intensive competition among many wireless service providers. However, given that there is little business research about the multi-tiered structure and the trade-off between competition and cooperation in the wireless service market, such a one-to-one model serves as an important first step. (In Appendix S9, we generalize our results to a competitive femtocell market, under the assumption that users have a fixed positive reservation payoff due to the existence of an alternative service.) A more general study on an oligopoly market is one of the interesting directions for future research.

We first examine the case in which the macrocell operator is the only service provider in the market. The operator determines its optimal service charge in order to maximize total profit. This case serves as a benchmark for our later analysis, in which a macrocell operator may lease partial resources to another operator to provide new femtocell service to the market.

The macrocell operator owns limited resources of wireless spectrum (also called bandwidth). Wireless users need to access the bandwidth in order to conduct their wireless communications (e.g., voice calls, video streaming, data transfer). A larger bandwidth means more resources for the user and thus better communication quality of service; however, it also means higher cost for the user.

We focus on the operation of a single macrocell and derive the optimal service charge under limited capacity. As shown in Figure 2, we model the interactions between the macrocell operator and end-users as a two-stage Stackelberg game. In Stage I, the macrocell operator determines the macrocell price p_M per unit bandwidth. Note that due to the exponential growth of wireless data traffic and the scarce spectrum resource, the usage-based pricing is becoming a main trend in the macrocell service market (Goldstein 2011). In Stage II, each user decides how much bandwidth to purchase. The users may be heterogeneous with their efficiencies in using wireless spectrum. The operator wants to maximize its profit, while the users want to maximize their individual payoffs. We solve

Figure 2 Two-Stage Stackelberg Game Between the Macrocell Operator and Users



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this two-stage Stackelberg game by backward induction (Myerson 2002, Duan et al. 2012).

3.1. Stage II: Users' Purchasing Decisions

The quality of service that a user receives depends not only on the size of the spectrum resource that he or she obtains, but also on the condition of the wireless channel. The channel condition is determined by both the location and the surrounding environment. For example, consider the transmissions from the users' mobile phones to the common single macrocell base station (as in Figure 1). The channel condition generally decreases with the distance between the user and the base station, and can become very weak if the user is inside a house with thick walls. A user with a bad channel condition will not be able to achieve a high data rate, even with a large bandwidth allocation.

We model the users' channel heterogeneity by a *macrocell spectrum efficiency* θ, which is assumed to be uniformly distributed in [0, 1]. A larger θ means a better channel condition and a higher spectrum efficiency *when using the macrocell service*. The uniform distribution is widely used to approximate wireless users' fluid population considering their locations (e.g., Altman et al. 2009, Inaltekin et al. 2007, and Leung and Huang 2011). This assumption also facilitates our analysis. However, we can show that using other continuous distributions, for example, normal distribution, will not change the main insights obtained in this study.

For a user with a macrocell spectrum efficiency $\theta < 1$, if the acquired macrocell bandwidth is b, then the user's effective resource allocation is θb. Similar to Chiang (2005), Huang et al. (2006), Sengupta and Chatterjee (2009), Song and Li (2005), and Wang and Li (2005), we model user's *utility* $u(\theta, b)$ as:

$$u(\theta, b) = \ln(1 + \theta b),$$

which is concave in b, representing the diminishing return in bandwidth consumption. Take watching a YouTube video as an example. It is very important to guarantee the basic level of bandwidth to support the video playback at the lowest quality of 360p. When the bandwidth allocation increases, a user can watch a higher resolution of the video (e.g., 480p, 720p, or even 1080p). However, the additional increase of satisfaction is sublinear with respect to the increase of data rate. The logarithmic function captures such a principle of diminishing returns. The "1" in the logarithmic function represents the fact that the user will experience zero benefit if the spectrum allocation is zero. The utility function can be interpreted as the data rate that the user obtains. The more bandwidth that a user acquires, the higher the data rate and better quality of service he or she receives (in the latest 4G cellular system based on OFDMA technology, different users communicating with the same macrocell or femtocell base station will be allocated different spectrum bands; Once the spectrum allocation is fixed, the communication performance among different users is decoupled and independent of each other). The product form of θ and b represents the fact that different users have different capabilities of using the spectrum resource. For an outdoor user who has a very good connection with the macrocell base station, he or she can fully utilize the bandwidth and achieve the maximum data transmission rate (the so-called "Shannon Capacity") in communications with proper coding and modulation schemes. However, for an indoor user behind thick walls, the connection between the cellular phone and the outdoor macrocell base station is weak, and the user obtains a much smaller data rate with the same amount of allocated bandwidth, compared with an outdoor user. A useful analogy to understand this is to imagine spectrum allocation b as the width of a road, and the spectrum efficiency θ as the type (speed) of the vehicle. How much traffic per unit time that the road can accommodate depends both on the width of the road and the types of the vehicles.

The user needs to pay a linear payment $p_M b$ to the macrocell operator, where the price p_M is announced by the macrocell operator in Stage I. The user's payoff is the difference between the utility and payment:

$$r_M(\theta, b, p_M) = \ln(1 + \theta b) - p_M b. \qquad (1)$$

The optimal amount of bandwidth that maximizes the user's payoff with the macrocell service is:

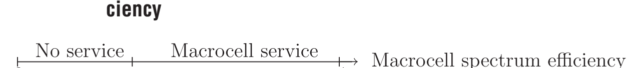
$$b^*(\theta, p_M) = \begin{cases} 1/p_M - 1/\theta, & \text{if } p_M \le \theta, \\ 0, & \text{if } p_M > \theta. \end{cases} \qquad (2)$$

$b^*(\theta, p_M)$ is decreasing in p_M and increasing in θ, when $p_M \le \theta$. As shown in Figure 3, the users are segmented into two groups – those who purchase the macrocell service and those who do not. Accordingly, a user's maximum payoff with macrocell service is:

$$r_M(\theta, b^*(\theta, p_M), p_M) = \ln\left(\frac{\theta}{p_M}\right) - 1 + \frac{p_M}{\theta}, \qquad (3)$$

if $p_M \le \theta$, and 0 otherwise.

Figure 3 Segmentation of Users Based on Macrocell Spectrum Efficiency



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3.2. Stage I: Macrocell Operator's Pricing p_M

Next, we consider the macrocell operator's optimal choice of price p_M in Stage I. To achieve a positive profit, the macrocell operator needs to set $p_M \leq \max_{\theta \in [0,1]} \theta = 1$; otherwise, no user will request any bandwidth in Stage II.

We denote the total user population as N. The fraction of users choosing macrocell service is $1 - p_M$, as shown in Figure 3. The total user demand is as follows:

$$Q_M(p_M) = N \int_{p_M}^{1} \left(\frac{1}{p_M} - \frac{1}{\theta} \right) d\theta = N \left(\frac{1}{p_M} - 1 + \ln p_M \right), \tag{4}$$

which is a decreasing function of p_M.

The macrocell operator has a limited bandwidth capacity B, and thus can only satisfy a total demand no larger than B. The macrocell operator optimizes p_M to maximize profit:

$$\max_{0 < p_M \leq 1} \pi^{macro}(p_M) = p_M \min \left(B, N \left(\frac{1}{p_M} - 1 + \ln p_M \right) \right). \tag{5}$$

Theorem 1 characterizes the unique optimal solution to Problem (5).

THEOREM 1. *The equilibrium macrocell price p_M^{bench} that maximizes the macrocell operator's profit in the two-stage Stackelberg game is the unique solution to the following equation:*

$$B = N \left(\frac{1}{p_M} - 1 + \ln p_M \right). \tag{6}$$

That is, the total user demand equals the maximum capacity at equilibrium.

It is easy to show that the equilibrium price p_M^{bench} decreases in B, and the macrocell operator's equilibrium profit $\pi^{macro}(p_M^{bench})$ increases in B. Furthermore, when the total bandwidth B is small, the equilibrium macrocell price p_M^{bench} becomes close to 1, and thus most users will not get service. This motivates the macrocell operator to consider facilitating the femtocell service, to serve more users and obtain a higher profit.

4. Femtocell Deployment

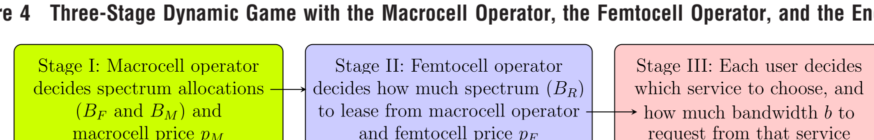
We now turn to the question of when and how femtocell service may improve the macrocell operator's profit. We are interested in understanding the following issues:

- *Strategic decision*: Is it economically beneficial for the macrocell operator to lease spectrum to a femtocell operator? How to evaluate the trade-off between cooperation and competition?
- *Operational decisions*: If the answer to the previous question is yes, how should the macrocell operator allocate and price the spectrum resources?

The analysis in this section is based on two simplified assumptions: (1) The femtocell service does not incur any additional operational cost compared to the macrocell service. This assumption will be relaxed in section 5. (2) The femtocell service has the same coverage as the macrocell service, so that users can freely choose between two services. The full femtocell coverage can be a good approximation when we look at a densely populated residential area with many deployed femtocells. This assumption will be relaxed in section 6.

We study a three-stage dynamic game, as shown in Figure 4. In Stage I, the macrocell operator decides bandwidth allocations to femtocell service B_F and macrocell service B_M, such that $B_F + B_M = B$. In this study, we consider the "separate carriers" scheme, where the macro and femtocell services operate on non-overlapping spectrum bands. Such a scheme is easy to manage and can avoid interferences between macrocells and femtocells. For example, China Unicom, one of the top three wireless service providers in China, has deployed femtocell service with this scheme since 2009 (China Femtocell Symposium 2011). The macrocell operator also decides the macrocell price p_M in Stage I, which is charged to both the femtocell operator and end-users who choose macrocell services. (In Appendix S6, we show that even if the macrocell operator is allowed to charge two different prices to the femtocell operator and the end-users, it is optimal for the macrocell operator to charge the same price to maximize his or her profit. Hence, we focus on the uniform pricing scheme in the study.)

Figure 4 Three-Stage Dynamic Game with the Macrocell Operator, the Femtocell Operator, and the End-Users



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In Stage II, the femtocell operator decides how much bandwidth B_R to lease from the macrocell operator, such that $B_R \leq B_F$. It also determines the femtocell retail price p_F. In Stage III, each user decides which service to choose and how much bandwidth to request. If a user's preferred service is not available, the user will seek the other service. We assume that there is a large group of users, where a single user's demand is infinitesimal to the total demand. Thus, we can ignore cases in which a user purchases bandwidth from both services.

Note that our model assumes that the macrocell operator is the leader and makes decisions first in Stage I, while the femtocell operator is the follower and reacts in Stage II. This is because, in practice, the macrocell operator is usually a market incumbent with much better network coverage and greater market power; whereas, the femtocell operator is usually a new market entrant with less market power (e.g., the Sprint-Virgin Mobile interaction in the United States and the BT-Vodafone interaction in the United Kingdom).

We analyze this three-stage game using backward induction. To distinguish from the benchmark case (macrocell service only) in section 3, we refer to the setup in this section as *dual services*. Note that dual services may degenerate to the benchmark case when the macrocell operator decides not to lease spectrum to the femtocell operator. Table 1 summarizes the key notation used in the model and analysis. All of the detailed proofs are provided in the Appendix.

4.1. Stage III: Users' Service Choices and Bandwidth Purchases

Given both macrocell and femtocell services, a user needs to decide which service to choose and how much bandwidth to request. If a user has a macrocell spectrum efficiency θ, his or her maximum payoff by using the macrocell service is given by Equation (3). Now we consider a user's payoff by using the femtocell service.

Table 1 Notation

p_M	The retail price charged to end-users by the macrocell operator
p_F	The retail price charged to end-users by the femtocell operator
p_W	The wholesale price charged to femtocell operator by the macrocell operator
B_R	The bandwidth requested by the femtocell operator from the macrocell operator
B_F	The bandwidth reserved by the macrocell operator for the femtocell operator
r_M	An end-user's payoff from using the macrocell service
r_F	An end-user's payoff from using the femtocell service
θ	The macrocell spectrum efficiency of an end-user, θ *in* [0, 1]
θ_{th}^{pr}	The threshold of end-users' preferred (initial) choices between the two services
θ_{th}	The threshold of end-users' final choices between the two services
π^{Macro}	The macrocell's total profit
π^{Femto}	The femtocell's total profit

Since femtocell base stations are deployed indoors and are very close to users' cell phones, it is reasonable to assume that all users using the femtocell service have equally good channel conditions and achieve the *maximum* spectrum efficiency 1. This means, independent of the macrocell spectrum efficiency θ, that each user achieves the *same* payoff $r_F(b, p_F)$ when using the femtocell service with a bandwidth of b:

$$r_F(b, p_F) = \ln(1 + b) - p_F b. \tag{7}$$

Accordingly, the optimal amount of bandwidth to request is:

$$b^*(p_F) = \frac{1}{p_F} - 1, \tag{8}$$

if $p_F \leq 1$ and 0 otherwise. A user's maximum payoff under the femtocell service is:

$$r_F(b^*(p_F), p_F) = \ln\left(\frac{1}{p_F}\right) - 1 + p_F, \tag{9}$$

if $p_F \leq 1$, and 0 otherwise.

We will show that $p_F > p_M$ at equilibrium, that is, the equilibrium femtocell price is always larger than the equilibrium macrocell price. By comparing the user's payoffs in Equations (3) and (9), it is clear that a user with $\theta = 1$ always chooses macrocell service to maximize the payoff. On the other hand, a user with a small θ would prefer to use femtocell service to improve the payoff. Let θ_{th}^{pr} denote the threshold of end-users' preferred (initial) choices between the macrocell and femtocell services. That is, users with $\theta \in [0, \theta_{th}^{pr})$ prefer to use the femtocell service, and users with $\theta \in [\theta_{th}^{pr}, 1]$ prefer to use the macrocell service.

Comparing a user's optimal payoff from using macrocell and femtocell services in Equations (3) and (9), we obtain the following lemma:

LEMMA 1. $\theta_{th}^{pr} = p_M/p_F$.

Because the macrocell and femtocell operators have limited bandwidth capacity, a user may not always obtain the preferred service. We assume that a user's service selection and payment process is as follows:

(i) Given prices of the femtocell and macrocell services, a user first computes the maximum payoff achievable under each service, and chooses the service with a higher payoff as his or her first (preferred) choice, or no service at all if the maximum payoffs are zero.

(ii) If the first choice request (e.g., the request to use femtocell service) is accepted, then the user would be charged based on the usage of that service. If the request is rejected, then the user can request the alternative service.

(iii) If the second choice request is accepted, then the user would be charged based on the usage of that service. If the second choice request is also rejected, then the user gets no service and pays nothing.

Next, we introduce the definition of *Final Demand*: If a user's demand from his or her preferred service is satisfied, then his or her final demand is the same as the preferred demand. Otherwise, the user would request the alternative service, and the new demand becomes the final demand. Note that a user's final demand may not always be satisfied, if the total final demand for the alternative service is larger than its spectrum capacity. Nevertheless, as the user is charged based on the actual service that he or she eventually receives, there is no uncertainty for the user.

When the macrocell operator receives more demand than its capacity, we assume that users with larger θ will be served first. This helps the macrocell operator to maximize the overall spectrum efficiency, as well as the revenue (see the proof in Appendix S1). It is also consistent with the current engineering practice, in which cell phones receiving stronger pilot signals from the macrocell base station will be served by the base station first.

Let θ_{th} denote the threshold of end-users' final choices between the macrocell and femtocell services. That is, users with $\theta \in [\theta_{th}, 1]$ receive the macrocell service, while users with $\theta \in [0, \theta_{th})$ receive either the femtocell service or no service. In general, θ_{th} may be different from θ_{th}^{pr}, depending on the service capacities. For example, a user who wants to subscribe to femtocell service may be rejected because the femtocell service is short of spectrum capacity. However, one of the key results that we will show later is that at equilibrium, all users' initial demands for their preferred services are satisfied, which means $\theta_{th}^{pr} = \theta_{th}$.

4.2. Stage II: Femtocell Operator's Leasing and Pricing Decisions

We now analyze Stage II, where the femtocell operator determines B_R and p_F to maximize its profit. In this stage, the macrocell operator's decisions on p_M and B_F (and $B_M = B - B_F$) are already determined and known to the femtocell operator. We denote the femtocell operator's equilibrium decisions as $B_R^*(p_M, B_F)$ and $p_F^*(p_M, B_F)$, which are functions of p_M and B_F.

To maximize profit, the femtocell operator needs to know which users will choose femtocell service and their characteristics. Users with macrocell spectrum efficiency $\theta \in [0, \theta_{th}^{pr})$ will choose femtocell service first. Some other users may also choose femtocell service if their preferred demands are not satisfied by the macrocell services. The following lemma, however, shows that the macrocell operator will reserve enough bandwidth B_M during Stage I, such that all users who prefer to use macrocell service will be satisfied.

LEMMA 2. *At equilibrium, the macrocell operator satisfies all preferred demands from users with* $\theta \in [\theta_{th}^{pr}, 1]$.

Since the femtocell operator only serves users with $\theta \in [0, p_M/p_F)$, its profit is:

$$\pi^{Femto}(p_F, B_R) = p_F \min\left(B_R, N\int_0^{\frac{p_M}{p_F}}\left(\frac{1}{p_F}-1\right)d\theta\right) - p_M B_R$$
$$= \min\left((p_F - p_M)B_R, N(1-p_F)\frac{p_M}{p_F} - p_M B_R\right). \quad (10)$$

The femtocell operator's profit-maximization problem is:

$$\max_{0<p_F\le 1, B_R\ge 0} \pi^{Femto}(p_F, B_R) \quad \text{subject to } B_R \le B_F. \quad (11)$$

By solving Problem (11), we have the following result:

LEMMA 3. *In Stage II, the femtocell operator's equilibrium femtocell price is:*

$$p_F^*(p_M, B_F) = \max\left(\frac{2p_M}{1+p_M}, \frac{-Np_M + \sqrt{(Np_M)^2 + 4NB_F p_M}}{2B_F}\right). \quad (12)$$

The equilibrium femtocell bandwidth purchasing amount is:

$$B_R^*(p_M, B_F) = \min\left(\frac{N(1-p_M^2)}{4p_M}, B_F\right), \quad (13)$$

which equals users' total preferred demand for femtocell service. As a result, $\theta_{th}^{pr} = \theta_{th}$.

4.3. Stage I: Macrocell Operator's Spectrum Allocation and Pricing Decisions

Now, we come back to Stage I, in which the macrocell operator determines the optimal p_M, B_F, and B_M to maximize its profit. Note that Lemma 2 shows that it is optimal for the macrocell operator to serve all users with $\theta \in [p_M/p_F^*(p_M, B_F), 1]$ by macrocell service, where $p_F^*(p_M, B_F)$ is the equilibrium femtocell price given in Lemma 2. This means that the macrocell operator does not want users with large macrocell spectrum efficiency θ to choose its competitor (i.e., the femtocell operator). Intuitively, users with a large θ demand more bandwidth in macrocell service than in femtocell service; thus, this leads to a larger profit for the macrocell operator if they choose macrocell service.

The macrocell operator's profit-maximization problem is:

$$\max_{0<p_M\le 1, B_F\ge 0} \pi^{Macro}(p_M, B_F) = p_M B_R^*(p_M, B_F) + N p_M \int_{\frac{p_M}{p_F^*(p_M,B_F)}}^{1} \left(\frac{1}{p_M} - \frac{1}{\theta}\right) d\theta. \quad (14)$$
$$\text{subject to } N \int_{\frac{p_M}{p_F^*(p_M,B_F)}}^{1} \left(\frac{1}{p_M} - \frac{1}{\theta}\right) d\theta \le B - B_F,$$

where $p_F^*(p_M, B_F)$ and $B_R^*(p_M, B_F)$ are given in Equations (12) and (13), respectively. The constraint states that the total macrocell demand cannot exceed the macrocell service capacity. Problem (14) is complex, and the following result can help us to simplify it.

LEMMA 4. *At equilibrium, we have* $B_F^* \le N(1 - (p_M^*)^2)/(4p_M^*)$, *which means that:*

$$B_R^*(p_M, B_F) = B_F,$$

$$p_F^*(p_M, B_F) = \frac{-Np_M + \sqrt{(Np_M)^2 + 4NB_F p_M}}{2B_F}.$$

Furthermore, we can show that the equilibrium macrocell price satisfies the following:

$$B_F + N \int_{\frac{p_M^*}{p_F^*(p_M^*,B_F)}}^{1} \left(\frac{1}{p_M^*} - \frac{1}{\theta}\right) d\theta = B.$$

Lemma 4 means that, at equilibrium, the macrocell operator will not reserve excessive bandwidth for the femtocell service, and the total spectrum demand equals the supply. With this, we can simplify Problem (14) to:

$$\max_{0<p_M\le 1, B_F\ge 0} \pi^{Macro} = p_M B$$
$$\text{subject to } B_F + N\left(\frac{1}{p_M} - \frac{p_M + \sqrt{p_M^2 + 4B_F p_M/N}}{2p_M} + \ln\left(\frac{p_M + \sqrt{p_M^2 + 4B_F p_M/N}}{2}\right)\right) = B, B_F \le \frac{N(1-p_M^2)}{4p_M}. \quad (15)$$

Next, we define y as:

$$y(p_M, B_F) = \frac{p_M + \sqrt{p_M^2 + 4B_F p_M/N}}{2}.$$

With further analysis (see Appendix S4), we can show that Problem (15) can eventually be simplified to the following one-variable optimization problem:

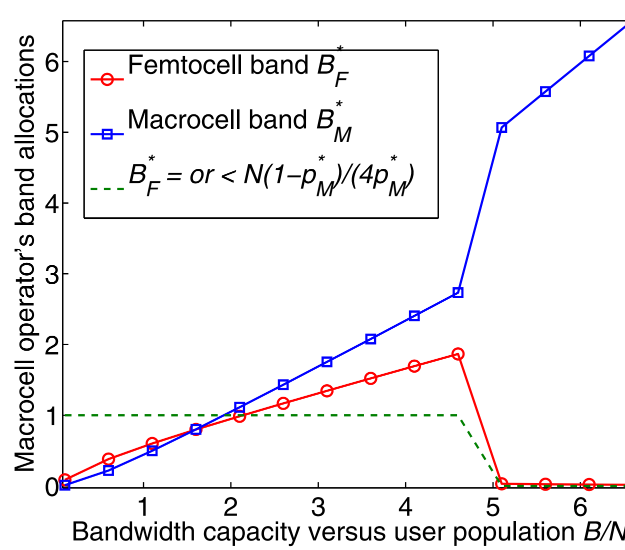
$$\max_y \pi^{Macro}(y) = B \cdot \frac{y^2 - y + 1}{y - \ln y + B/N}$$
$$\text{subject to } 2y - 1 - \frac{y^2 - y + 1}{y - \ln y + B/N} \le 0. \quad (16)$$

Problem (16) is still a non-convex optimization problem; thus, it is generally difficult to obtain the optimal solution in closed-form. Nevertheless, there are many efficient numerical methods to search the optimal solution. Hence, in the following, we use extensive numerical studies to demonstrate the equilibrium capacity allocation and pricing decisions.

4.4. Numerical Results

Figure 5 shows the macrocell operator's equilibrium bandwidth allocation. The x-axis is the total bandwidth capacity divided by the user population B/N, and the y-axis is the equilibrium bandwidth allocation. We also use a 1-0 binary value to denote whether or not the macrocell operator leases sufficient spectrum to the femtocell operator (the green curve). We see that when the bandwidth capacity is small, the macrocell operator would lease sufficient spectrum ($B_F^* = \frac{(1-p_M^{*})N}{4p_M^*}$) to the femtocell operator; however, when the total bandwidth capacity is large, only a

Figure 5 Equilibrium Bands B_F^* and B_M^* as Functions of Normalized Capacity B/N



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very small amount of bandwidth is allocated to the femtocell operator and most users are served via the macrocell service (i.e., B_F^* is close to zero and B_M^* is almost the same as B).

The intuition behind this is as follows: with large bandwidth capacity, the macrocell operator can already serve most users by macrocell service. Thus, the macrocell operator would lease only a very small amount of spectrum to the femtocell to serve users with very small θ. On the other hand, when the capacity is small, the macrocell service price p_M^* is high, and thus many users, with $\theta \in [0, p_M^*)$, would not request macrocell service. By leasing enough bandwidth to the femtocell operator, the macrocell operator can obtain a larger profit from serving more users (indirectly through the femtocell operator). Figure 5 shows that it is when B/N is around 4.77 that the change of the macrocell operator's leasing strategies occurs.

Figure 6 shows how the femtocell and macrocell prices p_F^* and p_M^* change in the total bandwidth capacity. It also shows the macrocell price of the benchmark case p_M^{bench} where there is macrocell service only. First, we observe that when the bandwidth capacity B/N becomes large, the femtocell price p_F^* becomes close to 1, and p_M^* approaches to p_M^{bench}. This essentially means that most users would be served by macrocell service (except for those with very small θ), which is consistent with the observation from Figure 5. Second, the equilibrium macrocell price p_M^* is always no less than the benchmark price p_M^{bench}. This means that the macrocell operator can obtain a larger profit with femtocell deployment.

Recall that $\theta_{th} = \frac{p_M^*}{p_F^*}$ is the equilibrium user segmentation threshold, i.e., users with $\theta \in [\theta_{th}, 1]$ choose the macrocell service, and the rest choose the femtocell service. We can show that if the macrocell operator had full control of both the macrocell and femtocell prices, then it would prefer the segmentation threshold to be $\tilde{\theta}_{th} = \frac{1}{\frac{1}{p_M^*}-\frac{1}{p_F^*}+1}$, i.e., users with $\theta \in \left[0, \frac{1}{\frac{1}{p_M}-\frac{1}{p_F^*(p_M, B_F)}+1}\right)$ choosing femtocell service, and the rest choosing macrocell service. The difference between these two thresholds is due to the market competition between the macrocell and femtocell operators. Figure 7 compares these two threshold values, and shows that the gap increases in the total capacity B/N initially, which means that the femtocell operator gradually attracts more users from macrocell service to femtocell service, and thus market competition becomes more intense. Nevertheless, when B/N becomes sufficiently large, only users with very small θ are served by the femtocell service, and the competition between the macrocell a nd femtocell service becomes much less.

By summarizing the results in Figures 5–7, we have the following observation:

Observation 1. *When the macrocell operator's total capacity is small, it will lease enough spectrum to the femtocell operator and thus increase the total users served by both operators. When the total capacity is large, the macrocell operator will only lease a very small amount of spectrum to the femtocell operator to reduce market competition.*

In practice, we observe that many macrocell operators in big cities do not have a large enough capacity to satisfy the fast growing wireless data demand, which leads to poor quality of service (LaVallee 2009,

Figure 6 p_F^* **and** p_M^* **in Dual Services, and Benchmark Price** p_M^{bench} **as Functions of** B/N

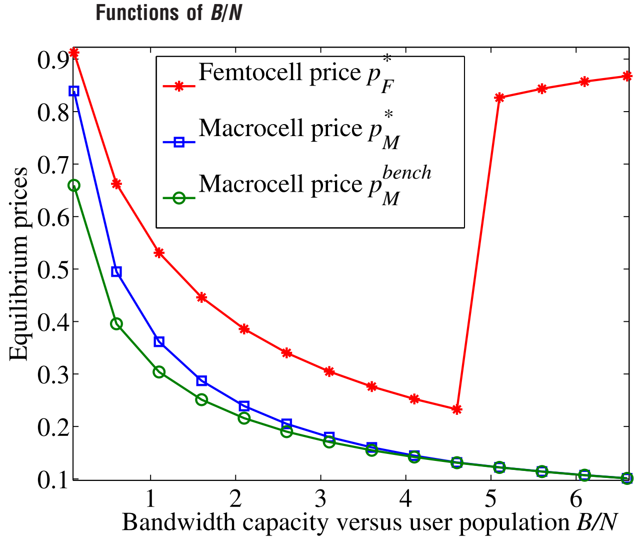
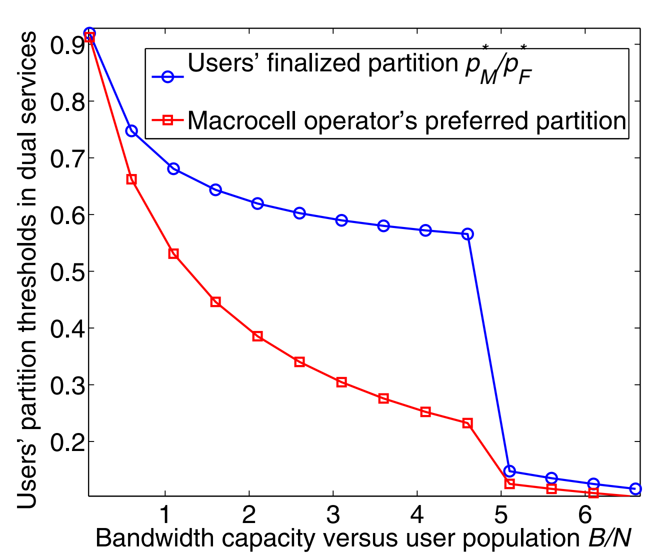


Figure 7 **The Equilibrium Final Threshold** $\theta_{th} = p_M^*/p_F^*$ **and the Macrocell Operator's Preferred** $\tilde{\theta}_{th} = \frac{1}{\frac{1}{p_M^*}-\frac{1}{p_F^*}+1}$ **as Functions of** B/N



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WNN Wi-Fi Net News 2008). One good example is the heavy congestion observed within the AT&T network in New York City and San Francisco, since the iPhone was introduced to the network in 2007. The recent "Global Mobile Data Traffic Forecast" published by Cisco in February 2013 has predicted that the global mobile data traffic will increase at an annual rate of 66% during the next few years, reaching 11.2 exabytes/month in 2017, a 13-fold increase compared to 2012. On the other hand, the global wireless spectrum available for macrocell networks only grew at an annual rate of 8% between 2007 and 2012. The spectrum will remain limited in the near future due to various physical constraints. This means that B/N will become relatively small and, as a result, we expect to see more femtocell services emerging.

Next, we investigate how the introduction of femtocell service affects the macrocell operator's profit, consumer surplus (i.e., users' aggregate payoff), and social welfare (i.e., summation of the profits of both operators and the payoffs of all users). In each figure, we compare the dual-service case with the macrocell-service-only benchmark case. Our discussions focus on the dual-service region.

Figure 8 shows the profits of the macrocell operator when both services are provided vs. when only macrocell service is provided. Both profits increase in the normalized capacity B/N. The provision of femtocell service can substantially increase the macrocell operator's profit, when the capacity is relatively small. When the capacity is sufficiently large, femtocell service is only used to serve users with very small θ; thus, the benefit to the macrocell operator is much less.

Figure 8 The Macrocell Operator's Profits in the Macrocell Service Only Case and the Dual Service Case as Functions of Capacity *B/N*

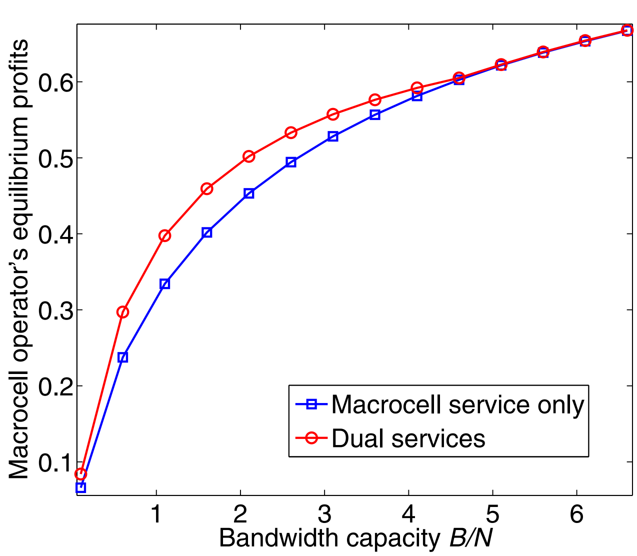


Figure 9 shows that the total consumer surplus increases with the deployment of femtocell service. This is mainly because users with low spectrum efficiency (small values of θ) will be able to obtain higher quality service by using femtocell. Nevertheless, some consumers could actually become worse off in the dual-service case. Figure 10 shows users' surplus (payoff) with respect to their macrocell spectrum efficiency θ, given the total capacity $B/N = 2.1$. It shows that a large θ user (e.g., $\theta > 0.6$) actually becomes worse off with the dual services. This is because the macrocell price increases under the dual service case, compared to the macrocell service only case. In

Figure 9 Comparison of Total Consumer Surplus Between Dual Services and Macrocell Service Only Benchmark as Functions of *B/N*

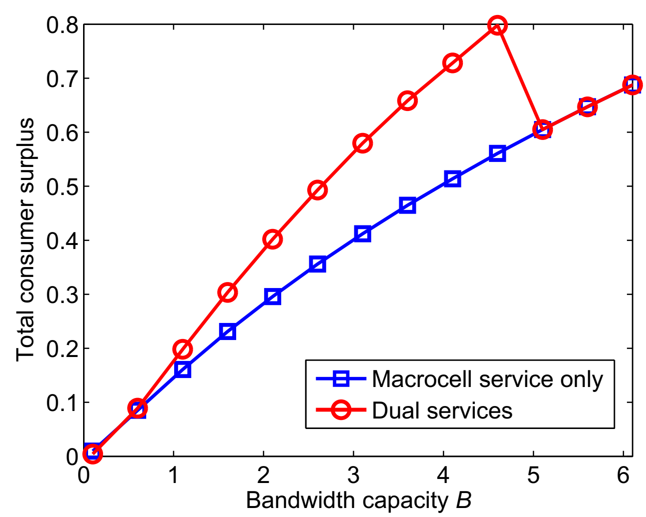
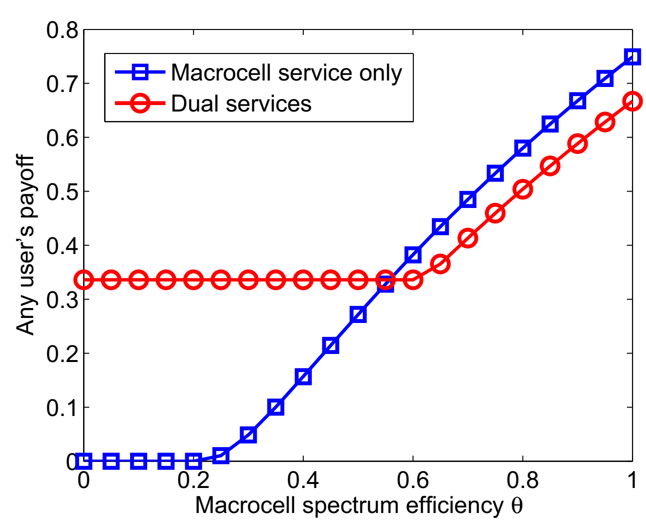


Figure 10 Comparison of a Consumer's Payoff Between Dual Services and Macrocell Service Only Benchmark as Functions of θ



contrast, users with very small θ benefit substantially from the dual service provision. Based on Figures 8–10, we have the following observation:

Observation 2. *After introducing the femtocell service, the macrocell operator's profit increases as more users receive service. Similarly, the total consumer surplus increases, although some users' payoffs can decrease. Overall, the total social welfare increases.*

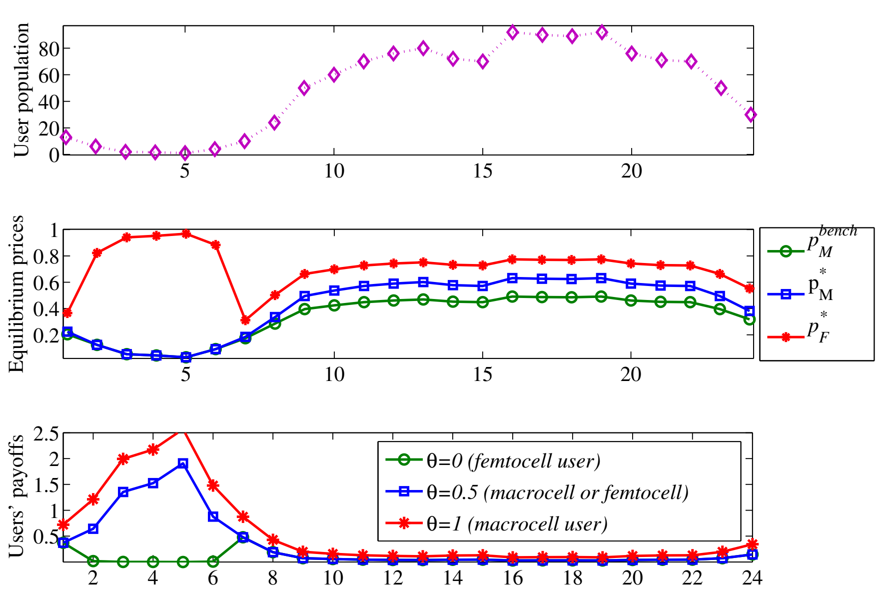
We also investigate how the decisions of the macrocell and femtocell operators are affected by the change of the active user number N. The first subfigure of Figure 11 demonstrates how the average number of end-users changes during 24 hours of a typical weekday, based on the measurements reported by Willkomm et al. (2009) (for proprietary reasons, Willkomm et al. (2009) show the relative user (call) arrival rates of 24 hours for every 5-minute interval, instead of the actual arrival rates; we approximate the arrival rates in a weekday using a one-hour interval and assume that the maximum arrival rate per hour is 100). The second subfigure shows how the service prices change in the day as the number of users changes, based on our analysis. Recall that p_M^{bench} is the macrocell service price in the macrocell-only scenario, and p_F^* and p_M^* are the equilibrium femtocell and macrocell prices in the dual services scenario. When N is small (between 1-6 am), the bandwidth capacity B is relatively adequate, in which case the macrocell operator will not lease much spectrum to the femtocell operator and p_F^* is close to 1. When N becomes larger (after 7 am), all prices become non-decreasing functions of N. Also, similar to Figure 6, the equilibrium price p_M^* in the dual service scenario is not smaller than the benchmark price p_M^{bench}. The last subfigure of Figure 11 shows that users' payoffs decrease with N when N is large, due to increased congestion level and higher prices.

5. Extension I: With Femtocell Operational Cost

In section 4, we consider a model in which femtocell service does not incur any additional cost compared with the macrocell service. As shown in Figure 1, femtocell users' traffic will first go through wireline broadband connection before reaching the control center of the cellular network. The broadband connection is owned by an Internet Service Provider (ISP). When the femtocell operator and the ISP belong to the same entity (e.g., both belonging to AT&T) or the ISP is sharing-friendly (NetShare Speakeasy 2002), there is no additional cost for broadband access. Otherwise, there is usually an access charge. In this section, we study the more general case in which the ISP charges the femtocell operator fees for using the wireline Internet connection in sending the femtocell users' traffic from the Internet. We examine how this operational cost affects the provision of femtocell service.

If the operation cost is a flat fee, then it is straightforward to see that the femtocell operator's equilibrium decisions in section 4 would remain unchanged if its revenue can compensate for the flat cost; otherwise, it would just not provide the femtocell service. In the following analysis, we assume that the total operational cost is linearly proportional to femtocell

Figure 11 Changes of User Population N, Equilibrium Prices (p_M^{bench} in Macrocell Only Service, and p_F^* and p_M^* in Dual Services), and Representative Users' Payoffs Throughout a Weekday



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bandwidth with a coefficient C. This is motivated by the fact that femtocell traffic uses ISP's broadband resource, and many ISPs have adopted usage-based pricing (Deleon 2011). Recall that in section 4 we have shown that a femtocell user's data rate and traffic volume are both linear in the demand for bandwidth; thus, femtocell users' aggregate traffic volume is linear in their total bandwidth demand (which equals the femtocell capacity).

We will focus on the case of $C \in (0, 1)$ in this section, since if $C \geq 1$, we can show that the femtocell operator will charge a femtocell price $p_F > C \geq 1$, and no user will choose the femtocell service. The three-stage decision process is similar to that shown in Figure 4. The analysis of Stage III is the same as in subsection 4.3, hence, we focus on Stages II and I in the following analysis. In this section we assume that the macrocell operator charges the same price to both the macrocell users and the femtocell operator. The analysis for the differentiated pricing setting is provided in Appendix S7.

In Stage II, the femtocell operator determines B_R and p_F to maximize its profit. We still use $B_R^*(p_M, B_F)$ and $p_F^*(p_M, B_F)$ to denote the equilibrium decisions of the femtocell operator. Without loss of generality, we normalize the value of N to 1. By following a similar analysis as of Lemma 2, we can obtain the following result:

LEMMA 5. *At equilibrium, the macrocell operator will satisfy all preferred demands from users with* $\theta \in [\theta_{th}^{pr}, 1]$.

Accordingly, the femtocell operator's profit-maximization problem is:

$$\max_{p_F \geq 0, B_R \geq 0} \pi^{Femto}(p_F, B_R) = (p_F - C)\min\left(B_R, \int_0^{\frac{p_M}{p_F}}\left(\frac{1}{p_F} - 1\right)d\theta\right) - p_M B_R \quad (17)$$

$$\text{subject to } B_R \leq B_F,$$
$$p_M + C \leq p_F \leq 1,$$

where the second constraint means that the femtocell price p_F should at least cover the total cost $(p_M + C)$ for the femtocell operator. Solving Problem (17), we have the following result:

LEMMA 6. *In Stage II, the femtocell operator's equilibrium femtocell price is:*

$$p_F^*(p_M, B_F) = \max\left(\frac{2}{1 + \frac{1}{p_M + C}}, \frac{-p_M + \sqrt{(p_M)^2 + 4B_F p_M}}{2B_F}\right), \quad (18)$$

and its equilibrium femtocell bandwidth purchase is:

$$B_R^*(p_M, B_F) = \min\left(p_M \frac{\frac{1}{(p_M + C)^2} - 1}{4}, B_F\right), \quad (19)$$

which equals users' total preferred demand for femtocell service. As a result, $\theta_{th} = \theta_{th}^{pr}$.

We now study Stage I, in which the macrocell operator determines p_M, B_F, and B_M to maximize its profit. The equilibrium decisions are denoted as p_M^*, B_F^*, and B_M^*. Lemma 5 states that the macrocell operator should serve all users with $\theta \in \left[\frac{p_M}{p_F^*(p_M, B_F)}, 1\right]$ by the macrocell service. The macrocell operator's profit-maximization problem is:

$$\max_{B_F \geq 0, p_M} \pi^{Macro}(p_M, B_F) = p_M B_R^*(B_F, p_M) + p_M \int_{\frac{p_M}{p_F^*(B_F, p_M)}}^{1}\left(\frac{1}{p_M} - \frac{1}{\theta}\right)d\theta \quad (20)$$

$$\text{subject to } 0 \leq B_F + \int_{\frac{p_M}{p_F^*(B_F, p_M)}}^{1}\left(\frac{1}{p_M} - \frac{1}{\theta}\right)d\theta \leq B,$$
$$0 < p_M \leq 1 - C,$$

where $B_R^*(B_F, p_M)$ and $p_F^*(B_F, p_M)$ are given in Equations (18) and (19), respectively. The second constraint shows that the total cost for the femtocell operator, $C + p_M$, should be less than 1; otherwise, no user will request the femtocell service.

5.1. Numerical Results

Problem (20) is not convex and is difficult to solve in closed-form, but can be solved easily using numerical methods. We are interested in investigating how the cost C affects the equilibrium capacity and pricing decisions.

We have shown in section 4 that, when the total spectrum capacity B is small, the amount of spectrum used for the macrocell and the femtocell services are comparable and both increase as B increases. However, when the macrocell's spectrum capacity reaches a certain threshold, the amount of spectrum leased to the femtocell operator drops dramatically. We define the corresponding value of spectrum capacity when this change occurs as *the boundary between the low capacity and high capacity regimes*.

Figure 12 shows how this boundary value changes with the femtocell operational cost. For example, the boundary value is 4.77 when C = 0 (also see Figure 5), and it becomes 3.58 when C = 0.1, and becomes 5.22 when C = 0.4. When C increases, the femtocell price p_F^* increases and the demand for femtocell service decreases. This makes it less attractive to provide femtocell service. On the other hand, the increase of price

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Figure 12 The Boundary Between Low and High Capacity Regimes Changes with Femtocell Operational Cost *C*

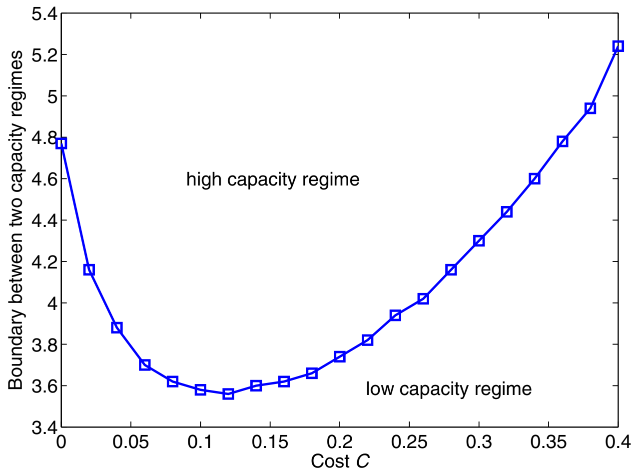


Figure 13 The Equilibrium Threshold $\theta_{th} = \frac{p_M^*}{p_F^*}$ and Macrocell Operator's Preferred Threshold $\tilde{\theta}_{th} = \frac{1}{\frac{1}{p_M^*}-\frac{1}{p_F^*}+1}$ with Respect to *C*

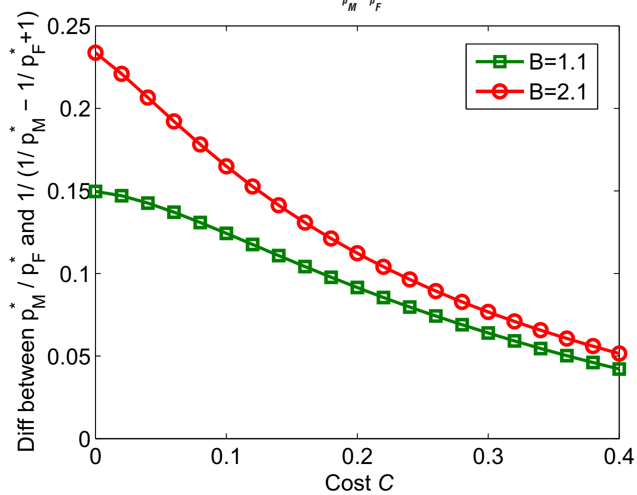
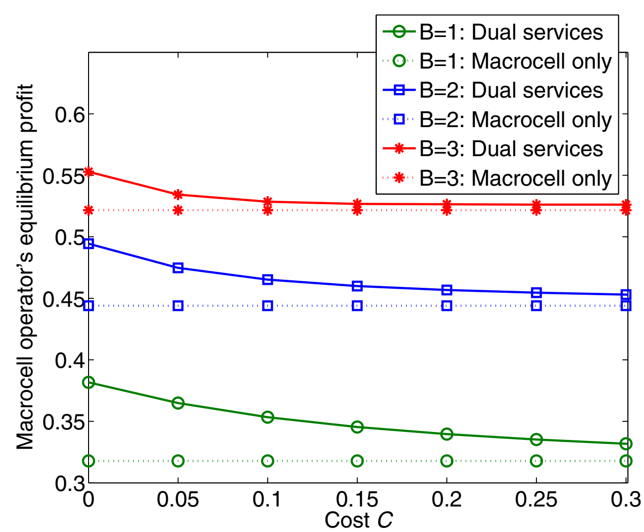


Figure 14 The Macrocell Operator's Equilibrium Profit as a Function of Cost *C* and Capacity *B*



p_F^* also reduces the market competition, which makes the macrocell operator more willing to lease spectrum to the femtocell operator. The interactions of these two factors determine the boundary of the two capacity regimes. More specifically, with a cost $C \leq 0.12$, the decrease of femtocell demands dominates, and the boundary decreases. With a larger cost $C > 0.12$, the decrease of competition dominates, and the boundary increases.

Figure 13 illustrates that, as C increases, the gap between the final threshold $\theta_{th} = \frac{p_M^*}{p_F^*}$ and the threshold $\tilde{\theta}_{th} = \frac{1}{\frac{1}{p_M^*}-\frac{1}{p_F^*}+1}$ that the macrocell operator prefers decreases, which means less service competition. Figures 14 and 15 show that the profits of the macrocell operator and the femtocell operator both decrease as the cost C increases. When C is smaller, the macrocell operator benefits more from enabling femtocell service; as C becomes larger, the benefit decreases and eventually the femtocell service becomes no longer economical and only the macrocell service is offered. Furthermore, for the macrocell operator, the benefit of having femtocell service is larger when the bandwidth capacity B is smaller, since under such scenarios the femtocell service can help the macrocell operator serve more users without introducing too much competition. This is consistent with what we observed when the cost C is zero.

6. Extension II: Limited Femtocell Coverage

In section 4, we assume that femtocell service has the same ubiquitous coverage as the macrocell service. In this section, we look at the general case in which the femtocell service only covers $\eta \in (0, 1)$ portion of the user population, and $1 - \eta$ portion of users can only access the macrocell service. We call the η fraction users *overlapped users*, and the rest $1 - \eta$ *non-overlapped users*. We are interested in understanding how the limited coverage affects the provision of femtocell service. In this setting, users only have macrocell service when they are out of femtocell coverage; however, they can still choose between the femtocell and macrocell services within the femtocell coverage. Thus, users' segmentation is characterized by macrocell spectrum efficiency θ and femtocell coverage η, and their decisions within the femtocell coverage areas are the same as those in section 4.

The three-stage decision process is similar to that depicted in Figure 4. The analysis of Stage III is the same as subsection 4.3. Without loss of generality, we normalize the user population N to 1. For Stage II,

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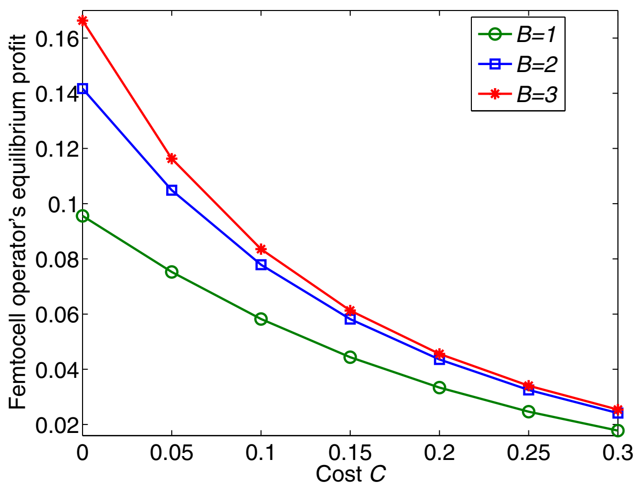
Figure 15 The Femtocell Operator's Equilibrium Profit as a Function of Cost *C* and Capacity *B*



following a similar analysis as of Lemma 2, we can also conclude that the overlapped users with $\theta \in [\theta_{th}^{pr}, 1]$ will be served by macrocell service, and the other overlapped users will be served by femtocell service, that is, $\theta_{th} = \theta_{th}^{pr}$. We derive the following result similar to Lemma 3:

LEMMA 7. *In Stage II, the femtocell operator's equilibrium femtocell price is:*

$$p_F^*(p_M,B_F)=\max\left(\frac{2}{\frac{1}{p_M}+1},\frac{-p_M\eta+\sqrt{(p_M\eta)^2+4p_MB_F\eta}}{2B_F}\right), \quad (21)$$

and its leased bandwidth from the macrocell operator is:

$$B_R^*(p_M,B_F) = \min\left(\eta p_M\left(\frac{\frac{1}{(p^M)^2}-1}{4}\right), B_F\right), \quad (22)$$

which equals overlapped users' total preferred demand in femtocell service.

Back to Stage I, the macrocell operator's profit-maximization problem is:

$$\max_{0<p_M\leq 1, B_F\geq 0} \pi^{Macro}(p_M,B_F)=p_MB_R^*(p_M,B_F)$$
$$+\eta p_M\int_{\frac{p_M}{p_F^*(p_M,B_F)}}^{1}\left(\frac{1}{p_M}-\frac{1}{\theta}\right)d\theta+(1-\eta)p_M\int_{p_M}^{1}\left(\frac{1}{p_M}-\frac{1}{\theta}\right)d\theta,$$
$$\text{subject to, } B_F+\eta\int_{\frac{p_M}{p_F^*(p_M,B_F)}}^{1}\left(\frac{1}{p_M}-\frac{1}{\theta}\right)d\theta$$
$$+(1-\eta)\int_{p_M}^{1}\left(\frac{1}{p_M}-\frac{1}{\theta}\right)d\theta\leq B, \quad (23)$$

where $p_F^*(p_M, B_F)$ and $B_R^*(p_M, B_F)$ are respectively given in Equations (21) and (22).

6.1. Numerical Results

Figure 16 shows that as η increases, it is more attractive to provide femtocell service, and the equilibrium femtocell (macrocell) band B_F^* (B_M^*) increases (decreases). Yet both prices p_F^* and p_M^* increase in η. The intuition is, as η increases, more users are served and the total femtocell demand increases, which leads to a larger p_F^*. The overall wireless service (macrocell plus femtocell) becomes more efficient, and the total user demand of both services increases, which leads to a larger p_M^*. Since the macrocell operator can sell its total capacity at a higher price, its profit increases in η. Fig 17 shows that the total consumer surplus in the

Figure 16 Equilibrium Prices p_F^* and p_M^* as Functions of η and *B*

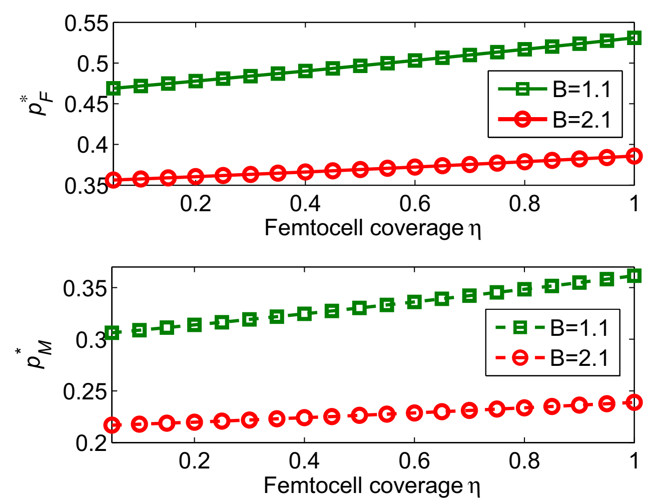
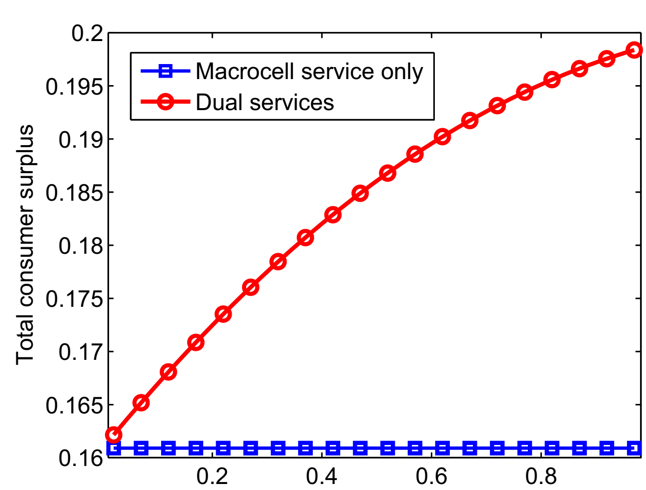


Figure 17 Comparison of Total Consumer Surplus Between Dual Services and Macrocell Benchmark Under *B* = 1.1



dual-service case increases in η, compared to the macrocell service only benchmark case.

OBSERVATION 3. *As femtocell coverage expands, the overall wireless service (macrocell plus femtocell) becomes more efficient. The macrocell operator's profit, total consumer surplus, and social welfare increase in η.*

7. Conclusion

This paper studies the economic incentives for a macrocell operator to deploy new femtocell service in addition to its existing macrocell service, by leasing spectrum to an independent femtocell operator. We model the interactions among the macrocell operator, the femtocell operator, and the end-users as a three-stage dynamic game.

We characterize the equilibrium decisions for the end-users' choices between the macrocell and the femtocell services, as well as the capacity allocation and pricing decisions for the macrocell and femtocell operators. Our analysis shows that the macrocell operator has more incentive to enable both macrocell and femtocell services when its total bandwidth is relatively tight, since femtocell service can help serve more users with small macrocell spectrum efficiency without introducing too much market competition. However, when the total bandwidth is large, femtocell service becomes a severe competitor to macrocell service, and thus the macrocell operator has less incentive to lease its bandwidth to the femtocell operator. In general, the introduction of femtocell service can increase the macrocell's profit, the total consumer surplus, and social welfare, although some macrocell users may experience a payoff decrease due to fewer resources and higher price. We also extend the main results to the more general cases, considering the additional femtocell operational cost and the limited femtocell coverage.

Our study represents an initial step to better understand the efficient management of new wireless services. There are several interesting directions for future research:

- We can further consider the "shared carriers" scheme besides "separate carriers," where femtocell service and macrocell service share part of or the whole spectrum. We can also consider frequency spectrum reuse, in which multiple femtocells can reuse the same spectrum if they do not overlap with each other. We need to optimize the pricing and spectrum allocation decisions by trading off the increased spectrum efficiency and mutual interferences between macrocell and femtocell services.
- We may study the case where both the macrocell and the femtocell services are provided by an integrated service provider, for example, a macrocell operator directly provides both macrocell and femtocell services to users, and it fully controls bandwidth resource allocation and service prices. We can also study the more general case of an oligopoly market, in which multiple macrocell/femtocell service providers compete with each other. Intuitively, we can envision that the macrocell and femtocell prices would go down due to new horizontal competitions. However, the technical analysis would be much more complex in the oligopoly case.
- We can consider the more general scenario in which a user travels through multiple locations over a long time horizon (e.g., over a year's time). If the macrocell and femtocell operators' prices are fixed at different locations, then a user's subscription decision in this general scenario can be decomposed into several static decision problems, as described in this study. However, if prices are dynamic, then it requests a new, comprehensive analysis to determine the optimal bandwidth allocation and pricing strategies, considering users' mobility patterns and spectrum efficiencies, as well as femtocell's deployment locations.

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References

Altman, E., A. Kumar, C. Singh, R. Sundaresan. 2009. Spatial SINR games combining base station placement and mobile association. *IEEE INFOCOM'09*.

Chambers, D. 2010. Opinions about Femtocell business case: Will BT demonstrate the femtocell MVNO business case? *ThinkSmallCell*. Available at http://www.thinksmallcell.com/Business-Case/will-bt-demonstrate-the-femtocell-mvno-business-case.html (accessed date July 1, 2015).

Chandrasekhar, V., J. G. Andrews. 2009. Uplink capacity and interference avoidance for two-tier femtocell networks. *Wireless Communications, IEEE Transactions* **8**(7): 3498–3509.

Chen, K., M. Kaya, Ö. Özer. 2008. Dual sales channel management with service competition. *Manuf. Serv. Oper. Manag.* **10**(4): 654–675.

Chen, Y., J. Zhang, Q. Zhang. 2012. Utility-aware refunding framework for hybrid access femtocell network. *IEEE Trans. Wirel. Commun.* **11**(5): 1688–1697.

Chiang, M. 2005. Balancing transport and physical layers in wireless multihop networks: Jointly optimal congestion control and power control. *IEEE J. Select. Areas Communi.* **23**(1): 104–116.

Chiang, W. K., D. Chhajed, J. D. Hess. 2003. Direct marketing, indirect profits: a strategic analysis of dual-channel supply-chain design. *Management Sci.* **49**(1): 1–20.

China Femtocell Symposium. 2011. Beijing. Available at http://www.conference.cn/femto/2011/en/Conference.asp?ArticleID=475 (accessed date July 1, 2015).

Claussen, H., L. Ho, L. Samuel. 2007. Financial analysis of a picocellular home network deployment. *IEEE ICC*.

David, A., E. Adida. 2015. Competition and coordination in a two-channel supply chain. *Prod. Oper. Manag.* **24**(8): 1358–1370.

Deleon, N. 2011. Usage-based billing hits Canada: say goodbye to Internet innovation. TechCrunch Hot Topics. Available at http://techcrunch.com/2011/02/01/usage-based-billing-hits-canada-say-goodbye-to-internet-innovation/ (accessed date July 1, 2015).

Duan, L., J. Huang, B. Shou. 2012. Duopoly competition in dynamic spectrum leasing and pricing. *IEEE Trans. Mobile Comput.* **11**(11): 1706–1719.

Duan, L., J. Huang, B. Shou. 2013. Economics of femtocell service provision. *IEEE Trans. Mobile Comput.* **12**(11): 2261–2273.

Dumrongsiri, A., M. Fan, A. Jain, K. Moinzadeh. 2008. A supply chain model with direct and retail channels. *Eur. J. Oper. Res.* **187**(3): 691–718.

eMarketer. 2014. Smartphone Users Worldwide Will Total 1.75 Billion in 2014. Available at http://www.emarketer.com/Article/Smartphone-Users-Worldwide-Will-Total-175-Billion-2014/1010536 (accessed date July 1, 2015).

Gabriel, C. 2009. Sprint to offer white label femto service to MVNOs. *Rethink Wireless*. Available at http://www.rethink-wireless.com/article.asp?article_id=1541 (accessed date July 1, 2015).

Goldstein, P. 2011. Verizon confirms it will ditch unlimited smartphone data plans starting July 7. *Fierce Wireless*. Available at http://www.fiercewireless.com/story/verizon-confirms-it-will-ditch-unlimited-smartphone-data-plans-starting-jul/2011-07-05 (accessed date July 1, 2015).

Huang, J., R. Berry, M. Honig. 2006. Distributed interference compensation for wireless networks. *IEEE J. Selected Areas Commun.* **24**(5): 1074–1084.

Huang, W., J. M. Swaminathan. 2009. Introduction of a second channel: Implications for pricing and profits. *Eur. J. Oper. Res.* **194**(1): 258–279.

Inaltekin, H., T. Wexler, S. B. Wicker. 2007. A duopoly pricing game for wireless IP services. *IEEE SECON'07*.

Informa Telecoms & Media. 2011. Femtocell deployments more than double in 12 months with strong growth byond the home. Available at http://www.informatm.com/itmgcontent/icoms/s/press-releases/20017850662.html (accessed date July 1, 2015).

Informa Telecoms & Media. 2012. Small-cell deployments to be dominated by consumer-driven femtocells.

LaVallee, A. 2009. AT&T to New York and San Francisco: we are working on it. *The Wall Street Journal*. Available at http://blogs.wsj.com/digits/2009/12/09/att-to-new-york-and-san-francisco-were-working-on-it/ (accessed date July 1, 2015).

Leung, K., J. Huang. 2011. Regulating wireless access pricing. *IEEE ICC'11*.

Myerson, R. B. 2002. *Game Theory: Analysis of Conflict*. Harvard University Press, Cambridge, MA.

NetShare Speakeasy. 2002. Available at http://www.prnewswire.com/news-releases/speakeasy-breaks-with-mainline-#isps-embraces-wi-fi-and-promotes-shared-broadband-connections-76903267.html (accessed date July 1, 2015).

Sandler, K. 2009. House calls: 'femtocells' promise to boost the cell phone signals inside your home. *The Wall Street Journal*. Available at http://www.wsj.com/articles/SB123447740993079445 (accessed date July 1, 2015)

Sengupta, S., M., Chatterjee. 2009. An economic framework for dynamic spectrum access and service pricing. *ACM/IEEE Trans. Network.* **17**(4): 1200–1213.

Shetty, N., S. Parekh, J. Walrand. 2009. Economics of femtocells. *IEEE GLOBECOM*.

Song, G. and Ye Li. 2005. Cross-layer optimization for OFDM wireless networks-part I: Theoretical framework. *IEEE Trans. Wirel. Commun.* **4**(2): 614–624.

Tsay, A. A., N. Agrawal. 2004a. Modeling conflict and coordination in multi-channel distribution systems: A review. In *Handbook of Quantitative Supply Chain Analysis*. Springer, US. 557–606.

Tsay, A. A., N. Agrawal. 2004b. Channel conflict and coordination in the e-commerce age. *Prod. Oper. Manag.* **13**(1): 93–110.

Wang, W., B. Li. 2005. Market-driven bandwidth allocation in selfish overlay networks. *IEEE INFOCOM*.

Willkomm, D., S. Machiraju, J. Bolot, A. Wolisz, 2009. Primary user behavior in cellular networks and implications for dynamic spectrum access. *IEEE Communications Magazine*.

WNN Wi-Fi Net News. 2008. Verizon wins big 700MHZ auction block. Available at http://wifinetnews.com/archives/2008/03/verizon_wins_big_700_mhz_auction_block.html (accessed date July 1, 2015).

Yun, S., Y. Yi, D. Cho, J. Mo. 2011. Open or close: on the sharing of femtocells. *IEEE INFOCOM Mini-Conference*.

Supporting Information

Additional Supporting Information may be found in the online version of this article:

Appendix S1: Proof of Lemma 2
Appendix S2: Proof of Lemma 3
Appendix S3: Proof of Lemma 4
Appendix S4: Further Anaylsis and Simplification of Problem (15) of the Main Paper
Appendix S5: Proof of Lemma 6
Appendix S6: Extension of Section 4 Results to Differentiated Pricing Case
Appendix S7: Differential Pricing Case with Femtocell Operational Cost
Appendix S8: Proof of Lemma 2 with Differentiated Pricing
Appendix S9: Femtocell Service Provision in a Competitive Market

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