Absolute instability in a traveling wave tube model

L. K. Ang and Y. Y. Lau

Department of Nuclear Engineering and Radiological Sciences, University of Michigan, Ann Arbor, Michigan 48109-2104

(Received 15 June 1998; accepted 27 July 1998)

A model is constructed to evaluate absolute instability which may lead to bandedge oscillations in a traveling wave tube. Under the assumptions (a) that all modes have forward group velocities, and (b) that the slow wave structure has a parabolic dispersion relation in the $\omega-k$ plane, the threshold coupling constant (Pierce’s parameter $C$) is calculated for the onset of absolute instability. The effect of distributed resistive loss in the circuit is included. The axial wave number and the characteristic frequency of the oscillation at the onset are given. © 1998 American Institute of Physics.

I. INTRODUCTION

There is a rekindling of interest in traveling wave tubes (TWTs), due to the recent advances in microwave power module and to the rapid growth in satellite communication, where TWTs are extensively used. In a TWT, the electron beam interacts with the forward wave of the slow wave circuit, resulting in spatial amplification of the signal. A serious threat to the stability of TWT is bandedge oscillation, characterized by a frequency in the vicinity of the $\pi$-mode of the slow wave circuit. A candidate for bandedge oscillation is the excitation of absolute instability, which may occur when all modes (beam and circuit) of interaction have forward group velocities. In this note, we present a calculation of this threshold current based on a simplified model of TWT.

In general, absolute instability can occur even if the TWT is perfectly matched at both ends. Thus, it is different from the regenerative oscillations caused by partial reflections of the amplified signal at the ends of the tube. It is also different from the backward wave oscillations that occur in the event that the beam mode intersects the circuit mode at a different from the backward wave oscillations that occur in the small signal regime [Fig. 1(b)]. We approximate the structure mode by a parabola $\omega = \omega_{BE} - \alpha(k-k_{BE})^2$, where $k_{BE} = \pi/L$ is the wave number at bandedge, $\omega_{BE}$ is the band-edge frequency, and $\alpha$ determines the “shape” of the parabola. When $v = v_0$, the beam mode dispersion curve is tangent to the parabola at $\omega = \omega_0$, $k = k_0$ [Fig. 1(b)]. We call this case, $v = v_0$, the synchronous case. In terms of $k_{BE}$, $\omega_{BE}$ and $\alpha$, we find $k_0 = (k_{BE}^2 - \omega_{BE}^2/\alpha)^{1/2}$, $\omega_0 = 2\alpha(k_{BE} - k_0)/k_0$, and $v_0 = \omega_0/k_0$. Note that for $v > v_{BE}$, where $v_{BE} = \omega_{BE}/k_{BE}$, all interacting modes have forward group velocities [Fig. 1(b)]. In this paper, we concentrate only on such cases.

By normalizing $\omega$ with respect to $\omega_0$ and $k$ to $k_0$, we model the TWT dispersion relation as

$$D(\bar{k}, \bar{\omega}) = (\bar{k}(1 + \delta - \bar{\omega}^2)[P(\bar{k}^2 - 1)^2 - (\bar{k}^2 - \bar{\omega}^2) + \Delta(1-j)] = \bar{\epsilon},$$

(1)

$$P = \frac{k_0}{2(k_{BE} - k_0)}.$$  

(2)

Here, $\bar{\omega} = \omega/\omega_0$, $\bar{k} = k/k_0$, and $\epsilon > 0$ ($\epsilon =$ current) is the dimensionless coupling constant. In Eq. (1), the first square bracket represents the beam mode and the second square bracket represents the structure mode, when these square brackets are set equal to zero individually. We have included the effects of beam detuning through $\delta = (v - v_0)/v_0$ (either positive or negative) in the beam mode, and of a distributed resistive loss through $\Delta(>0)$ in the structure mode. We take...
the dimensionless parameters \( \delta \) and \( \Delta \) to be small, and \( 0 < k_0/k_{BE} < 1 \). In terms of \( P \), we find \( k_{BE} = k_0/1 + 1/2P \).

We then obtain \( \varepsilon_s = D(\bar{k}_s, \bar{\omega}_s) \), which reads, for a lossless structure,

\[
\varepsilon_s = \frac{27(1 + \delta)^6 \Lambda^4}{P^3},
\]

where

\[
\bar{k}_s = 1 + \frac{1}{2P} \left[ 1 - 2 \Lambda (1 + \delta) \right],
\]

\[
\bar{\omega}_s = (1 + \delta) \left[ 1 + \frac{1}{2P} \left( 1 - 8 \Lambda (1 + \delta) \right) \right],
\]

It is obvious from Eq. (3) that \( \varepsilon_s \) decreases with increasing \( P \). This occurs when \( k_0 \) approaches the bandedge wave number \( k_{BE} \) [cf. Eq. (2)]. Note from Eqs. (6) and (3) that when \( \delta = -1/2 (1 + 2P) \), \( \Lambda = 0 \) and \( \varepsilon_s \) becomes zero. This case of zero threshold current may be shown to correspond to the beam mode intersecting the structure mode at the bandedge \((v = v_{BE})\) at various \( \delta \), the corresponding frequency of oscillation at the onset is shown in Fig. 2(b). From Fig. 2(a), we see that the threshold increases (decreases) with positive (negative) \( \delta \), and becomes very small as \( k_0 \) approaching \( k_{BE} \).

The analysis for a lossy structure \((\Delta > 0)\) is more involved. Here, \( \bar{\omega}_s \) remains real and \( \bar{k}_s \) becomes complex at the onset of absolute instability. In this case, we first find a relation \( \bar{k} = \bar{k}(\bar{\omega}) \) from the condition \( \partial D/\partial \bar{k} = 0 \).
of \( \bar{\omega} \), this function \( \vec{k}(\bar{\omega}) \) is in general complex; so is \( \varepsilon = D(\vec{k}(\bar{\omega}), \bar{\omega}) \). The critical value of \( \varepsilon \) is obtained for that real value of \( \bar{\omega} \) at which \( \text{Re}[D(\vec{k}(\bar{\omega}), \bar{\omega})]=0 \) and \( \text{Im}[D(\vec{k}(\bar{\omega}), \bar{\omega})]=0 \). We have checked that in the limit \( \Delta = 0 \) (lossless case), this numerical procedure yields the same results as those derived analytically for the lossless case given above. In Fig. 3, \( \varepsilon^{1/3}_s \) for \( \delta = 0 \) is shown as a function of \( k_0/k_{BE} \) at various \( \Delta \). Here, we see that the presence of loss (\( \Delta > 0 \)) raises the threshold current for absolute instability. The stabilization of the bandedge oscillation is more effective at larger \( k_0/k_{BE} \). In Fig. 4, the dependence of \( \varepsilon^{1/3}_s, \bar{\omega}_s \) and \( \vec{k}_s \) (real and imaginary part) on \( k_0/k_{BE} \) is shown for the case \( \Delta = 0.001 \) and \( \delta = 0 \).

The above analysis may offer some guidance on improving the stability of TWT, in terms of the required distributed loss and, more importantly, of the separation between the \( \pi \)-mode frequency and the amplifying band. The analytic solution provides a scaling on the gain parameter. Because of the sensitivity to the parameter \( P \), which is a measure of the separation between the synchronous mode and the \( \pi \)-mode wave numbers [cf. Eq. (2)], \( P \) needs to be assigned empirically for the tube under study.

III. CONCLUDING REMARKS

Finally, we note some differences between absolute instability in fast wave TWT and in slow wave TWT. In a fast wave TWT, such as the gyrotron\(^6\) or gyropeniotron,\(^7\) absolute instability always occurs near the waveguide cutoff (with zero group velocity), at which the axial wave number is close to zero. Thus, absolute instability in a fast wave tube is quite sensitive to the axial length of the tube, and may not be as readily detected.\(^4\) On the other hand, absolute instability in a slow wave TWT occurs near the \( \pi \)-mode (also with zero group velocity\(^7\)), whose axial wavelength is much shorter than the axial length of the tube. Thus, absolute instability in a conventional TWT is much less sensitive to the length of the tube, and is much more readily observed.

ACKNOWLEDGMENTS

We wish to thank David Chernin, Carter Armstrong, Mark Basten, Baruch Levush, and Mark Baird for many useful discussions.

This work was supported by the Multidisciplinary University Research Initiative (MURI), managed by the Air Force Office of Scientific Research and subcontracted through Texas Tech University, by the Northrop Grumman Industrial Associates Program, and by the Naval Research Laboratory.


\(^7\)Strictly speaking, for a perfect helix, the circuit mode does not have a zero group velocity at the \( \pi \)-mode wavenumber \( \pi/L \). See, e.g., Fig 2.9 of D. A. Watkins, Topics in Electromagnetic Theory (Wiley, New York, 1958). However, any slight imperfection in the helix, which is unavoidable in practice, will make \( k_{BE} = \pi/L \) a zero group velocity wavenumber. We wish to thank David Chernin for pointing this out to us. Carter Armstrong has also kindly informed us that oscillations at the \( \pi \)-mode are of serious concern in the design of TWT.
