SIMULATION OF STANDING WAVES USING MOVING-GRID TECHNIQUES WITH SPECTRAL ELEMENT METHODS

R. Bouffanais[†,*], **M. O. Deville**[†]
[†], STI-ISE-LIN, École Polytechnique Fédérale de Lausanne, Station 9, CH-1015 Lausanne, Switzerland
*Email: roland.bouffanais@epfl.ch

Abstract

We present the numerical techniques developed for simulating nonlinear standing waves in a rectangular basin above incompressible and viscous fluid flows using spectral element methods (SEM). In this moving-boundary context, our technique relies on moving-grid techniques in the arbitrary Lagrangian-Eulerian (ALE) framework aiming at maintaining the spectral convergence and accuracy.

Introduction

Spectral element methods [5,6] provide an efficient and accurate approach to simulating many physical problems. The vast majority of applications using SEM for the spatial discretization of the governing equations deal with relatively simple geometries with fixed boundaries. In this work we present a novel technique based on SEM capable of simulating problems involving relatively complex geometries with moving-boundaries. Based on a moving-grid technique in the ALE framework [2, 3, 7], the numerical simulations of two- and three-dimensional nonlinear standing waves in a rectangular basin have been carried out, showing very good results compared to available theoretical ones.

These numerical results will be presented together with the details of the numerical technique and the validation test cases ensuring that the exponential rate of convergence characterizing SEM is maintained.

Presentation details

A brief introduction will be devoted to recalling the physical problem and its Navier-Stokes variational formulation in the framework of the ALE kinematic description coupled with the moving-grid governing equation [1]. Following that, the spectral element discretization based on a classical Galerkin formulation will be derived and associated with a discretization of the kinematic boundary condition for the free surface [1]. A decoupled approach has been considered for the moving-grid algorithm and for the ALE Navier-Stokes algorithm. Particular emphasis has been put on the moving-grid technique that relies on calculations of the mesh velocity inside the fluid domain. For consistency with our physical problem and with SEM, our choice of mesh update strategy is based on considering the mesh motion as a steady incompressible viscous fluid motion governed by the steady Stokes equations. The major advantage of this choice is to ensure the incompressibility of the mesh, leading to a valuable conservation of the volume of the spectral elements used in the computation of the ALE Navier-Stokes problem for the fluid. The viscous diffusive part of the semidiscrete equations of the problem is treated implicitly whereas the nonlinear convective part, which involves the mesh velocity and the moving-mesh equation are dealt with explicitly. Influence of the moving-grid technique on the global time-integration order will be discussed.

Validation results will be presented and discussed for different motions of the grid imposed via the choice of an analytical expression of the mesh velocity. These results are showing the accuracy and reliability of our code even for large-amplitude motions of the inner nodes of the mesh and of the boundaries (see Figure 1).

The second half of the presentation will focus on analyzing the behaviour of our moving-grid SEM code and the results for the numerical simulations of the two-dimensional monochromatic nonlinear standing waves and three-dimensional nonlinear standing waves in a rectangular basin above an incompressible and Newtonian fluid flow [4]. The initial free-surface shape is imposed analytically from the theoretical expression given by the nonlinear theory of finite amplitude standing waves. Simulations for different values of the Reynolds number have been conducted. In the three-dimensional case, the initial free-surface position is computed from two identical standing waves that are orthogonally superposed. These two initially independent waves interact and give rise to several modes showing, as expected, a viscous damping that is analyzed.

Conclusions

The flexibility and the spectral rate of convergence offered by the afore-introduced moving-grid technique using SEM make it very well designed for handling nonlinear waves as shown in the series of results that have been presented. These fundamental features of our numerical technique allow us to investigate more arduous and complex problems such as waves interacting with underneath turbulent flows.

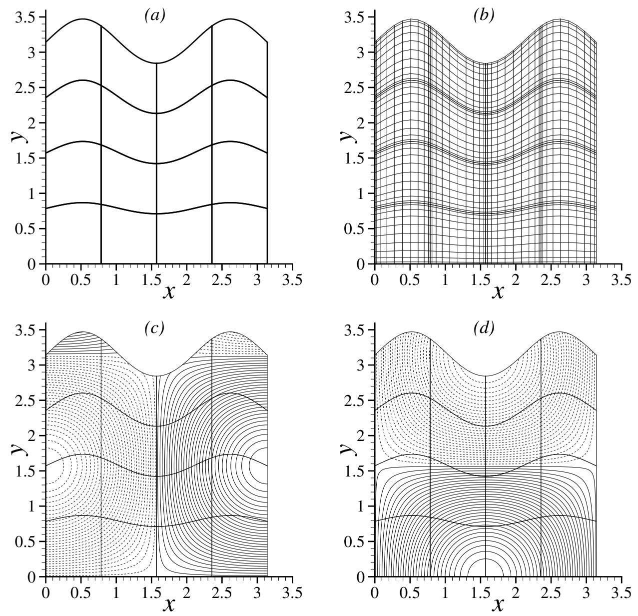

Figure 1: Simulations for the code validation based on the flow of decaying vortices [4]; Results at 5 time units starting with a non-deformed squared domain $[0,\pi]^2$: (a) Spectral elements decomposition of the domain; (b) Deformed mesh generated by its motion; (c) (resp. (d)) Contour lines of the first (resp. second) component of the velocity field.

References

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