Large-eddy simulation of the flow in a lid-driven cubical cavity using dynamic approximate deconvolution models

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Abstract :

LES of the flow in a lid-driven cubical cavity by the Legendre spectral element method using a novel dynamic approximate deconvolution model (ADM) is considered. Explicit filtering is based on an invertible modal filter. Despite its simple geometry, this 3D unsteady flow at $\mathrm{Re} = 12'000$ *is very challenging for subgrid modeling. Indeed, maintaining the energy balance among scales in such a confined fluid domain is a difficult task allowing to track any under- or over-dissipative character of the subgrid dynamic ADM.*

Résumé :

LES de l'écoulement dans une cavité cubique entrainée par la méthode des éléments spectraux et utilisant un nouveau modèle de déconvolution approchée est considérée. Le filtrage explicite est basé sur un filtre modal inversible. Malgré la simplicite de la géométrie, cet écoulement instationnaire à $\mathrm{Re} = 12\,000$ *pose de problèmes en termes de modélisation de sous-maille. En effet, maintenir l'équilibre énergétique entre les différentes échelles de l'écoulement est une tâche difficile pour un tel écoulement confiné.*

Key-words :

turbulence; large-eddy simulation; lid-driven cavity flow

1 Introduction

Approximate deconvolution models (ADM) constitute a particular family of subgrid models. They rely on the attempt to recover, at least partially, the original unfiltered fields, up to the grid level, by inverting the filtering operator applied to the Navier–Stokes equations. The focus here is on the approximate iterative method introduced by Stolz and Adams (1999) which is based on the van Cittert procedure.

LES of Newtonian incompressible fluid flows with ADM based on the van Cittert method using the Legendre spectral element method (SEM) as spatial discretization to solve the filtered Navier–Stokes equations are envisaged for the first time in this work. A novel subgrid model which blends ADM and the mixed scale model introduced by Sagaut (1996) with a dynamic evaluation of the subgrid-viscosity constant based on a Germano–Lilly type of procedure is proposed.

A DNS of the flow in a lid-driven cubical cavity performed at Reynolds number of 12'000 with a Chebyshev collocation method due to Leriche and Gavrilakis (2000) is taken as the reference solution to validate the new model. Subgrid modeling in the case of a flow with coexisting laminar, transitional and turbulent zones such as the lid-driven cubical cavity flow represents a challenging problem. As the flow is confined and recirculating, any under- or over-dissipative character of the subgrid model can be clearly identified. Moreover, the very low dissipation and dispersion induced by SEM allows a pertinent analysis of the energetic action induced by any subgrid model, which is not feasible in the framework of low-order numerical methods. The coupling of the lid-driven cubical cavity flow problem with the SEM builds therefore a well suited framework to analyze the accuracy of the newly defined subgrid model.

2 Governing equations and numerical method

In the case of isothermal flows of Newtonian incompressible fluids, the LES governing equations for the filtered quantities denoted by an *overbar*, are obtained by applying a convolution filter $\mathcal{G}\star$ to the Navier–Stokes equations. The filtered velocity field $\overline{\mathbf{u}} = \mathcal{G} \star \mathbf{u}$ satisfies a divergence-free condition through the filtered reduced pressure field $\overline{p}$. The closure of the filtered momentum equation requires the subgrid tensor $\boldsymbol{\tau} = \overline{\mathbf{u}\mathbf{u}} - \overline{\mathbf{u}}\,\overline{\mathbf{u}}$, to be expressed in terms of the filtered field which reflects the subgrid scales modeling and the interaction among all space scales of the solution.

2.1 Space discretization

The numerical method treats the filtered Navier–Stokes equations within the weak Galerkin formulation framework. In each spectral element, the velocity and pressure fields are approximated using Lagrange–Legendre polynomial interpolants. The reader is referred to the monograph by Deville *et al.* (2002) for full details. The velocity and pressure are expressed in the $\mathbb{P}_p - \mathbb{P}_{p-2}$ functional spaces where $\mathbb{P}_p$ is the set of polynomials of degree lower than p in each space direction. This spectral element method avoids the presence of spurious pressure modes as it was proved by Maday and Patera (1989). The quadrature rules are based on a Gauss–Lobatto–Legendre (GLL) grid for the velocity nodes and a Gauss–Legendre grid (GL) for the pressure nodes.

2.2 Time integration

Standard time integrators in the SEM framework handle the viscous linear term and the pressure implicitly by a backward differentiation formula of order 2 (BDF2), while all nonlinearities, including the discretized subgrid term, are computed explicitly, e.g. by a second order extrapolation method (EX2), under a CFL restriction. The implicit part is solved by a generalized block LU decomposition with a pressure correction algorithm. The overall order-in-time of the afore-presented numerical method is two.

3 Approximate deconvolution model

3.1 General considerations

The deconvolution approach aims at reconstructing the unfiltered fields from the filtered ones. The subgrid modes are not modeled but reconstructed using an *ad hoc* mathematical procedure. Writing formally the Navier–Stokes momentum equation as

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{f}(\mathbf{u}) = \mathbf{0}, \tag{1}$$

the evolution equation of the filtered quantities becomes

$$\frac{\partial \overline{\mathbf{u}}}{\partial t} + \mathbf{f}(\overline{\mathbf{u}}) = [\mathbf{f}, \mathcal{G}\star](\mathbf{u}), \tag{2}$$

where the convolution filter $\mathcal{G}\star = (\mathcal{L} \circ \mathcal{P})\star$ embodies the LES filter $\mathcal{L}\star$ and the projective grid filter $\mathcal{P}\star$, the latter being therefore implicitly accounted for in the sequel. The subgrid commutator reads then

$$[\mathbf{f}, \mathcal{G}\star](\mathbf{u}) = \mathbf{f}(\mathcal{G} \star \mathbf{u}) - \mathcal{G} \star \mathbf{f}(\mathbf{u}) = \mathbf{f}(\overline{\mathbf{u}}) - \overline{\mathbf{f}(\mathbf{u})} = -\boldsymbol{\nabla} \cdot \boldsymbol{\tau}, \tag{3}$$

The exact subgrid contribution appears as a function of the non-filtered field, which is not computed when performing a LES. This field being unknown, the idea is to approximate it using the following deconvolution procedure

$$\mathbf{u} \simeq \mathbf{u}^* = \mathcal{Q}_N \star \overline{\mathbf{u}} = (\mathcal{Q}_N \circ \mathcal{G}) \star \mathbf{u} = (\mathcal{Q}_N \circ \mathcal{L} \circ \mathcal{P}) \star \mathbf{u} = (\mathcal{Q}_N \circ \mathcal{L}) \star \hat{\mathbf{u}}, \tag{4}$$

where $\hat{\mathbf{u}} = \mathcal{P} \star \mathbf{u}$ is the grid-filtered velocity. The operator $\mathcal{Q}_N\star$ is an Nth-order approximation of the inverse of the filter $\mathcal{L}\star$, since the grid filter is projective and therefore not invertible. Stolz and Adams (1999) proposed an iterative deconvolution procedure based on the van Cittert method. If the filter $\mathcal{L}\star$ has an inverse, it can be computed using the truncated van Cittert expansion series expressed with the identity operator $\mathcal{I}\star$

$$\mathcal{L}^{-1} \simeq \mathcal{Q}_N = \sum_{i=0}^{N} (\mathcal{I} - \mathcal{L})^i. \tag{5}$$

The subgrid term is then approximated as

$$[\mathbf{f}, \mathcal{G}\star](\mathbf{u}) = -\boldsymbol{\nabla} \cdot \boldsymbol{\tau} \simeq [\mathbf{f}, \mathcal{G}\star](\mathcal{Q}_N \star \overline{\mathbf{u}}) = [\mathbf{f}, \mathcal{G}\star](\mathbf{u}^*), \tag{6}$$

leading to the following expression of the filtered Navier–Stokes momentum equation

$$\frac{\partial \overline{\mathbf{u}}}{\partial t} + \mathbf{f}(\overline{\mathbf{u}}) = [\mathbf{f}, \mathcal{G}\star](\mathbf{u}^*). \tag{7}$$

3.2 Coupling with a dynamic mixed scale model

Coupling ADM with a subgrid-viscosity model can be formally achieved by adding a source term $\mathrm{s}(\overline{\mathbf{u}})$ to the right-hand side of Eq. (7)

$$\frac{\partial \overline{\mathbf{u}}}{\partial t} + \mathbf{f}(\overline{\mathbf{u}}) = [\mathbf{f}, \mathcal{G}\star](\mathbf{u}^*) + \mathrm{s}(\overline{\mathbf{u}}), \tag{8}$$

where $\mathrm{s}(\overline{\mathbf{u}})$ is expressed in terms of the filtered rate-of-strain tensor $\overline{\mathrm{S}}$ by

$$\mathrm{s}(\overline{\mathbf{u}}) = \boldsymbol{\nabla} \cdot (\nu_{\mathrm{sgs}}(\boldsymbol{\nabla}\overline{\mathbf{u}} + \boldsymbol{\nabla}\overline{\mathbf{u}}^T)) = \boldsymbol{\nabla} \cdot (2\nu_{\mathrm{sgs}}\overline{\mathrm{S}}), \tag{9}$$

the superscript 'T' denoting the transpose operation and ν_{sgs} the subgrid viscosity.

In the sequel, we focus on a subgrid-viscosity model proposed by Sagaut (1996) having a triple dependency on the large and small structures of the resolved field, and the filter cutoff length $\overline{\Delta}$. With respect to the Smagorinsky model used by Winckelmans *et al.* (2001), the model proposed by Sagaut offers the advantage of automatically vanishing if subgrid scales are absent of the solution. This model, which makes up the one-parameter mixed scale family, is derived by taking a weighted geometric average of the models based on large scales and those based on the energy at cutoff. The closure is given by

$$\nu_{\mathrm{sgs}} = C_\gamma |\mathbf{S}(\overline{\mathbf{u}})|^\gamma (\overline{q}_{\mathrm{c}})^{\frac{1-\gamma}{2}} \overline{\Delta}^{1+\gamma}, \tag{10}$$

where C_γ and γ are the subgrid-viscosity and mixed-scale constants, $\overline{q}_{\rm c}$ is the resolved kinetic energy at cutoff which can be evaluated using the formula

$$\overline{q}_{\rm c} = \frac{1}{2}\overline{u}_{{\rm c},i}\overline{u}_{{\rm c},i}, \tag{11}$$

where the cutoff velocity field $\overline{\mathbf{u}}_{\rm c}$ represents the high-frequency part of the resolved field, defined using a second filter, referred to as test filter, designated by the *tilde* symbol and associated with the cutoff length $\widetilde{\Delta} > \overline{\Delta}$

$$\overline{\mathbf{u}}_{\rm c} = \overline{\mathbf{u}} - \widetilde{\overline{\mathbf{u}}}. \tag{12}$$

We note that for $\gamma \in [0, 1]$, the subgrid viscosity is always defined. The constant C_γ can be evaluated by theories of turbulence in the case of statistically homogeneous and isotropic turbulent flow

$$C_\gamma = C_{\rm q}^{1-\gamma} C_{\rm s}^{2\gamma}, \tag{13}$$

where the Smagorinsky constant $C_{\rm s} \simeq 0.18$ and $C_{\rm q} \simeq 0.20$.

Theoretical values of the subgrid-viscosity constant cannot be used in our case because they are derived if the model is used without the ADM structural contribution, that is to model the whole subgrid tensor. In order to overcome this issue, we introduce a dynamic procedure of Germano–Lilly type to evaluate this parameter as a function of space and time. Such procedure completes the definition of the subgrid model based on the coupling of ADM with the dynamic mixed scale (DMS) model, called ADM-DMS in the sequel.

4 Filtering

Filtering techniques suited to SEM and LES must preserve C^0-continuity of the filtered variables across spectral elements and be applicable at the element level. In the sequel, the filter used satisfies these constraints. The filtering operation is performed by applying a given transfer function in a modal basis. Depending on this transfer function, this filter may not be projective, therefore ensuring its invertibility which is a key feature needed by the deconvolution procedure. Full details on this specific filtering technique are given in Habisreutinger *et al.* (2007).

5 LES of the lid-driven cubical cavity flow

The different LES presented hereafter refer to the flow in a lid-driven cubical cavity performed at Reynolds number of 12'000. Although the geometry is very simple, the flow presents complex physical phenomena as described by Leriche and Gavrilakis (2000), no direction of homogeneity and a large variety of flow conditions. The origin of the axes is located at the geometrical center, the x-axis (resp. y-axis) being horizontal (resp. vertical) and the z-axis is in the transverse direction. For such Reynolds numbers, the flow over most of the domain is laminar and turbulence develops near the cavity walls. Its main feature is a large scale recirculation which spans the cavity in the z-direction. Aside this large flow structure, the relatively high momentum fluid near the lid is deviated by the downstream wall into a down flowing nonparallel wall jet which separates ahead of the bottom wall. A region of high pressure and dissipation located at the top of downstream wall results from this deviation. The energy resulting from the impingement of the separated layer against the bottom wall is lost to turbulence and partly recovered by an emerging wall jet near the upstream wall where the flow slows down and relaminarizes during the fluid rise. The flow is also characterized by multiple counter-rotating recirculating regions at the corners and edges of the cavity.

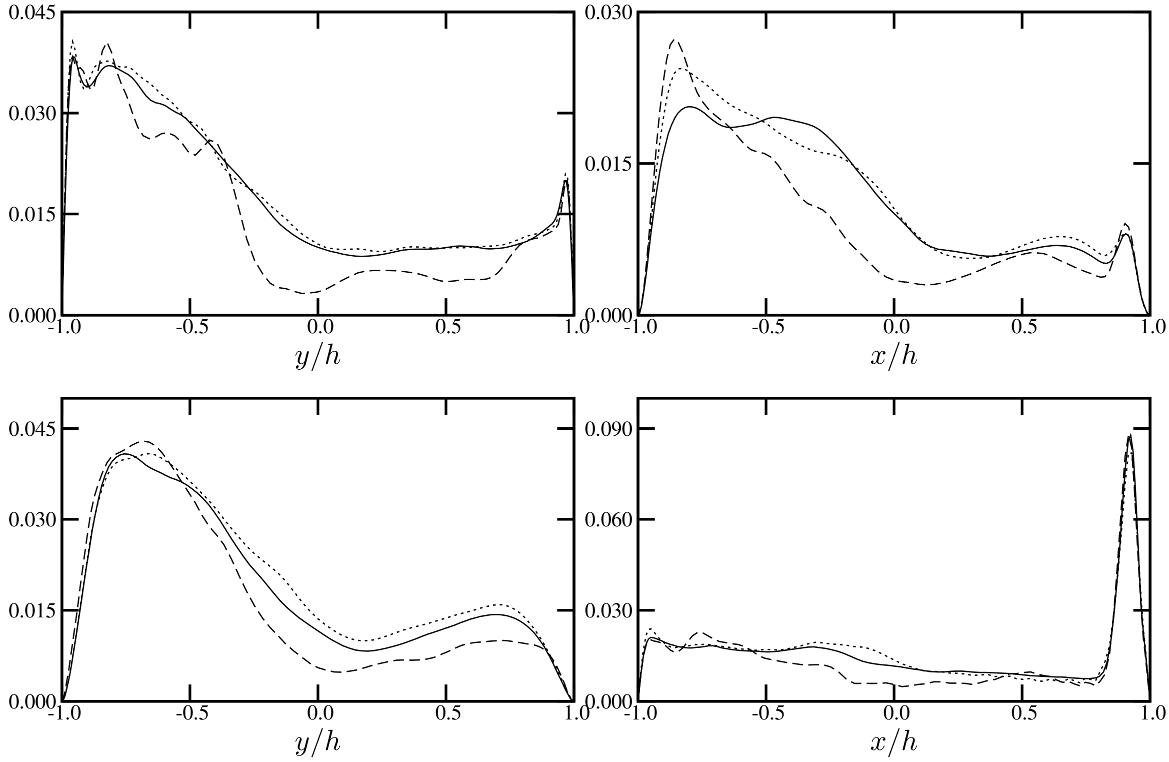

Figure 1: In the mid-plane $z/h = 0$, DMS (dashed lines), ADM-DMS (dotted lines) and DNS (solid lines). Top row: u-rms, bottom row: v-rms. Left column: on the horizontal centerline $x/h = 0$, right column: on the vertical centerline $y/h = 0$.

The physical and numerical parameters of the DNS and the LES are the same as Leriche and Gavrilakis (2000) and Bouffanais *et al.* (2007) respectively. The DNS constitutes the reference solution and was obtained with a Chebyshev collocation method on a grid composed of 129 collocation points in each spatial direction Leriche and Gavrilakis (2000). For LES, the spectral elements are unevenly distributed in order to resolve the boundary layers along the lid and the downstream wall. The spatial discretization has $E_x = E_y = E_z = 8$ elements in the three space directions with $p_x = p_y = p_z = 8$ polynomial degree, equivalent to 65^3 grid points in total. The mesh used for LES has therefore twice less points per space direction than the DNS grid of Leriche and Gavrilakis. A LES based on DMS with the same parameters as ADM-DMS for its dynamic mixed scale part, is also presented and compared to ADM-DMS in order to identify the improvement induced by coupling ADM with DMS.

The mixed scales constant is set to $\gamma = 0.5$ in order to have the triple dependency on the large and small structures of the resolved field as a function of the filter cutoff length. The choice of the deconvolution order is based on the observations of Stolz *et al.* (2001) who found that the value $N = 5$ for the deconvolution order is a good compromise between the precision in the approximate deconvolution and the computational cost induced by higher N in the van Cittert expansion series.

The comparisons with the DNS results are performed by plotting one-dimensional plots of u-rms and v-rms on the vertical and horizontal centerlines of the mid-plane of the cavity. Figure 1 showcases the improvement achieved in terms of subgrid modeling by coupling ADM with DMS. Indeed, the variations of u-rms and v-rms for ADM-DMS reproduce quite accurately the intense-fluctuations zones in the mid-plane $z/h = 0$. DMS appears clearly not as effective as ADM-DMS.

6 Conclusions

LES of Newtonian incompressible fluid flows with ADM based on the van Cittert method using Legendre-SEM have been performed. A coupling with a dynamic mixed scale model was introduced. The coupling of the lid-driven cubical cavity flow problem at Reynolds number of 12'000 with the SEM having very low numerical dissipation and dispersion appears to be a well suited framework to analyze the accuracy of the proposed subgrid model.

Accounting for the reduced sampling and integration time, the LES performed with ADM-DMS show good agreement with the reference results. More precisely, first- and second-order statistics are in good agreement when compared to their DNS counterparts. Results for the Reynolds stresses production, coupling first- and second-order statistical moments, are also well predicted using this new model even with such reduced sampling. The analysis of the results obtained with DMS allows us to clearly identify the improvement induced by coupling ADM with DMS.

All the presented results emphasize the efficiency of ADM-DMS when dealing with laminar, transitional and turbulent flow conditions such as those occurring in the lid-driven cubical cavity flow at Re $= 12'000$.

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